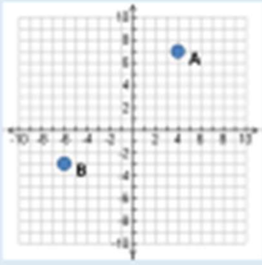
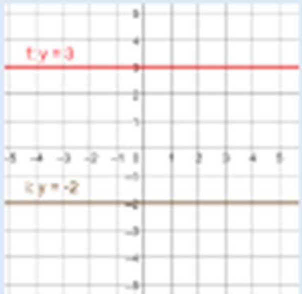
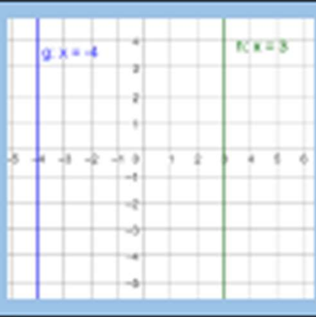
Maths

LET ALL OUR BRIGHT COLOURS SHINE

Year 11 Knowledge Organisers

Year 11 Graphs…

Straight Line Graphs

Topic/Skill	Definition/Tips	Example
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across, left or right**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Midpoint of a Line	Method 1: **add the x coordinates and divide by 2, add the y coordinates and divide by 2** Method 2: Sketch the line and find the values half way between the two x and two y values.	Find the midpoint between (2,1) and (6,9) $\frac{2+6}{2} = 4$ and $\frac{1+9}{2} = 5$ So, the midpoint is (4,5)
3. **Horizontal Lines**	The equation of a **horizontal line** is **y = a**, where a is a number For example the graph **y = 3** is a **horizontal line** with the **y co-ordinate of 3**	
4. **Vertical Lines**	The equation of a **vertical line** is **x = a**, where a is a number For example the graph **x = - 4** is a **vertical line** with the **x co-ordinate of - 4**	
5. Linear Graphs with slope	**Straight line** graph. The general equation of a linear graph is $y = mx + c$ where m **is the gradient** and c **is the y-intercept**. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$

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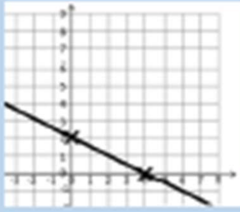
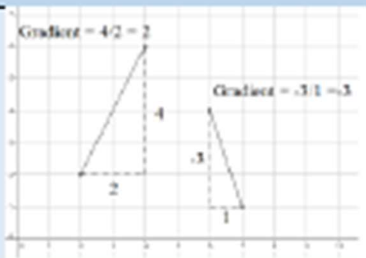
<table>
<tr><td>6. Plotting Linear Graphs</td><td>Method 1: Table of Values
Construct a table of values to calculate coordinates.

Method 2: Gradient-Intercept Method (use when the equation is in the form $y = mx + c$)
1. Plots the y-intercept
2. Using the gradient, plot a second point.
3. Draw a line through the two points plotted.

Method 3: Cover-Up Method (use when the equation is in the form $ax + by = c$)
1. Cover the x term and solve the resulting equation. Plot this on the $x - axis$.
2. Cover the y term and solve the resulting equation. Plot this on the $y - axis$.
3. Draw a line through the two points plotted.</td><td><table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y= x +3</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>
$2x + 4y = 8$</td></tr>
<tr><td>7. Gradient</td><td>The gradient of a line is how steep it is.

Gradient =
$$\frac{Change\ in\ y}{Change\ in\ x} = \frac{Rise}{Run}$$
The gradient can be positive (sloping upwards) or negative (sloping downwards)</td><td></td></tr>
<tr><td>8. Finding the Equation of a Line given a <u>point and a gradient</u></td><td>Substitute in the gradient (m) and point (x,y) in to the equation $y = mx + c$ and solve for c.</td><td>Find the equation of the line with gradient 4 passing through (2,7).
$$y = mx + c$$ $$7 = 4 \times 2 + c$$ $$c = -1$$ $$y = 4x - 1$$</td></tr>
<tr><td>9. Finding the Equation of a Line <u>given two points</u></td><td>Use the two points to calculate the gradient. Then repeat the method above using the gradient and either of the points.</td><td>Find the equation of the line passing through (6,11) and (2,3)
$$m = \frac{11-3}{6-2} = 2$$ $$y = mx + c$$ $$11 = 2 \times 6 + c$$ $$c = -1$$ $$y = 2x - 1$$</td></tr>
<tr><td>10. Parallel Lines</td><td>If two lines are parallel, they will have the same gradient. The value of m will be the same for both lines.</td><td>Are the lines $y = 3x - 1$ and $2y - 6x + 10 = 0$ parallel?</td></tr>
</table>

Year 11 Graphs…

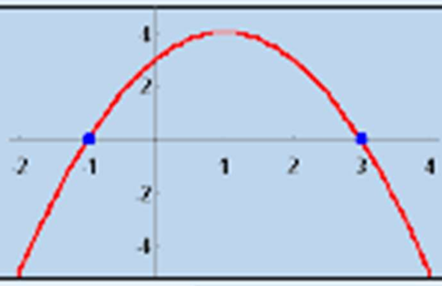
Straight Line Graphs

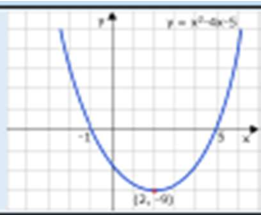
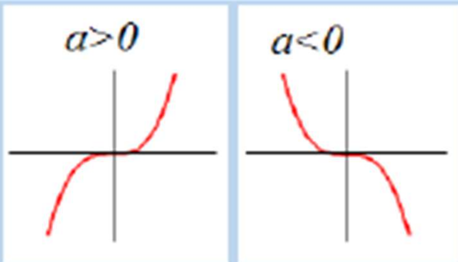
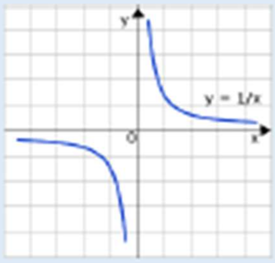
		Answer: Rearrange the second equation in to the form $y = mx + c$ $2y - 6x + 10 = 0 \rightarrow y = 3x - 5$ Since the two gradients are equal (3), the lines are parallel.
11. Perpendicular Lines	If two lines are **perpendicular**, the **product** of their **gradients** will always equal **-1**. The gradient of one line will be the **negative reciprocal** of the gradient of the other line. You may need to rearrange equations of lines to compare gradients (they need to be in the form $y = mx + c$)	Find the equation of the line perpendicular to $y = 3x + 2$ which passes through (6,5) Answer: As they are perpendicular, the gradient of the new line will be $-\frac{1}{3}$ as this is the negative reciprocal of 3. $y = mx + c$ $5 = -\frac{1}{3} \times 6 + c$ $c = 7$ $y = -\frac{1}{3}x + 7$ Or $3x + x - 7 = 0$

6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. **The solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$

Year 11 Graphs…

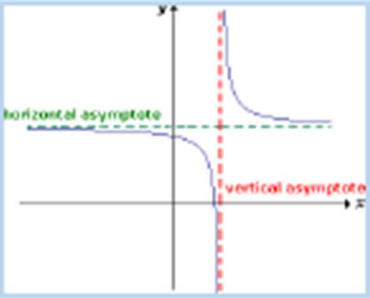
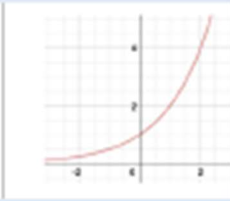
Non-Linear Graphs

9. Quadratic Graph	A 'U-shaped' curve called a **parabola**. The equation is of the form $y = ax^2 + bx + c$, where a, b and c are numbers, $a \neq 0$. If $a < 0$, the parabola is **upside down**.	
10. Roots of a Quadratic	A **root** is a **solution**. The roots of a quadratic are the x-**intercepts of the quadratic graph**.	
11. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	

3. Quadratic Graph	A 'U-shaped' curve called a **parabola**. The equation is of the form $y = ax^2 + bx + c$, where a, b and c are numbers, $a \neq 0$. If $a < 0$, the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $y = ax^3 + k$, where k is an number. If $a > 0$, the curve is increasing. If $a < 0$, the curve is decreasing.	
5. Reciprocal Graph	The equation is of the form $y = \frac{A}{x}$, where A is a number and $x \neq 0$. The graph has asymptotes on the x-axis and y-axis.	

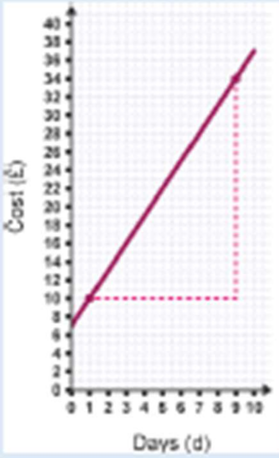
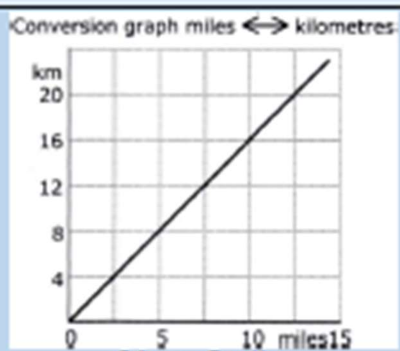
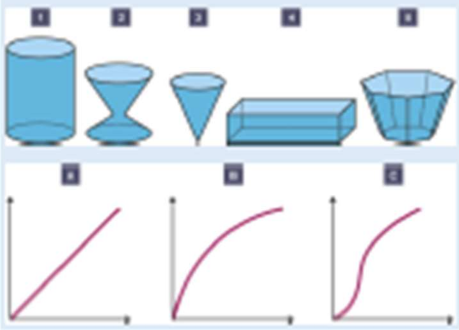
Year 11 Graphs…

Non-Linear Graphs

6. Asymptote	A **straight line** that a graph **approaches** but **never touches.**	
7. Exponential Graph	The equation is of the form $y = a^x$, where a is a number called the base. If $a > 1$ the graph increases. If $0 < a < 1$, the graph decreases. The graph has an asymptote which is the **x-axis.**	 

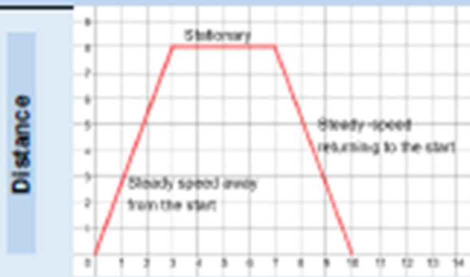
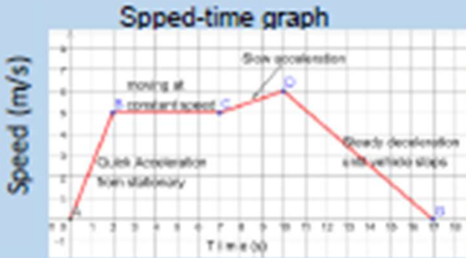
Year 11 Graphs…

Real-Life Graphs

Topic/Skill	Definition/Tips	Example
1. Real Life Graphs	Graphs that are supposed to model some real-life situation. The actual meaning of the values depends on the labels and units on each axis. The **gradient** might have a contextual meaning. The **y-intercept** might have a contextual meaning. The **area** under the graph might have a contextual meaning.	 A graph showing the cost of hiring a ladder for various numbers of days. The gradient shows the cost per day. It costs £3/day to hire the ladder. The y-intercept shows the additional cost/deposit/fixed charge (something not linked to how long the ladder is hired for). The additional cost is £7.
2. Conversion Graph	A line graph to **convert one unit to another**. Can be used to convert units (eg. miles and kilometres) or currencies ($ and £) Find the value you know on one axis, read up/across to the conversion line and read the equivalent value from the other axis.	 $8\ km = 5\ miles$
3. Depth of Water in Containers	Graphs can be used to show how the depth of water changes as different shaped containers are filled with water at a constant rate.	

Year 11 Graphs…

Real-Life Graphs

Topic/Skill	Definition/Tips	Example
4. Distance time graphs	The distance is on the y – axis while time is on the x – axis. A positive **gradient** gives the speed when movement is away from the starting point. A negative **gradient** give the speed when movement is towards the starting point. The **horizontal** part of the graph means the object is **stationary**.	
5. Speed time graphs	The speed is on the y – axis while time is on the x – axis. A positive **gradient** gives the acceleration or rate of increase of speed. A negative **gradient** give the deceleration or rate of decrease of speed The **horizontal** part of the graph means moving at constant speed. The area under the graph gives the total distance travelled.	

Year 11 Algebra…

Expanding & Factorising

9. Expand	To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket. To expand two brackets, **multiply** each term in **one bracket** by each term in **the other bracket** and then simplifying terms if possible	$3(m+7)=3x+21$ $(a+b)(c+d)=ac+ad+bc+bd$ $(x+2)(x+4)=x^2+4x+2x+8$ $=x^2+6x+8$
10. Factorise	**The reverse of expanding.** Factorising is writing an expression as a product of terms by **'taking out' a common factor.**	$6x-15=3(2x-5)$, where 3 is the common factor.

1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2+bx+c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2-3x+7$ Examples of non-quadratic expressions: $2x^3-5x^2$ $9x-1$
2. Factorising Quadratics	When a quadratic expression is in the form x^2+bx+c find the two numbers **that add to give b and multiply to give c.**	$x^2+7x+10=(x+5)(x+2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2+2x-8=(x+4)(x-2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2-b^2}$ can be factorised to give $(\boldsymbol{a+b})(\boldsymbol{a-b})$	$x^2-25=(x+5)(x-5)$ $16x^2-81=(4x+9)(4x-9)$
4. Solving Quadratics ($ax^2=b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution.**	$2x^2=98$ $x^2=49$ $x=\pm 7$
5. Solving Quadratics ($ax^2+bx=0$)	Factorise and then solve = 0.	$x^2-3x=0$ $x(x-3)=0$ $x=0$ or $x=3$
6. Solving Quadratics by Factorising ($a=1$)	Factorise the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2+3x-10=0$ Factorise: $(x+5)(x-2)=0$ $x=-5$ or $x=2$
7. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form ax^2+bx+c 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2+5x-4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2+8x-3x-4$ 4. Factorise in pairs: $2x(3x+4)-1(3x+4)$ 5. Answer = $(3x+4)(2x-1)$
8. Solving Quadratics by Factorising ($a \neq 1$)	Factorise the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2+7x-4=0$ Factorise: $(2x-1)(x+4)=0$ $x=\frac{1}{2}$ or $x=-4$

Year 11 Algebra…

Rearranging Formulae

3. Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz+1}{2} = x$ We now have x as the subject.

Year 11 Algebra…

Functions

Topic/Skill	Definition/Tips	Example
1. Function Machine	Takes an **input** value, performs some **operations** and produces an **output** value.	
2. Function	A **relationship** between two sets of values.	$f(x) = 3x^2 - 5$ 'For any input value, square the term, then multiply by 3, then subtract 5'.
3. Function notation	$f(x)$ x is the **input** value $f(x)$ is the **output** value.	$f(x) = 3x + 11$ Suppose the input value is $x = 5$ The output value is $f(5) = 3 \times 5 + 11 = 26$
4. Inverse function	$f^{-1}(x)$ A function that performs the **opposite process** of the original function. 1. Write the function as $y = f(x)$ 2. Rearrange to make x the subject. 3. Replace the y **with** x and the x **with** $f^{-1}(x)$	$f(x) = (1 - 2x)^5$. Find the inverse. $y = (1 - 2x)^5$ $\sqrt[5]{y} = 1 - 2x$ $1 - \sqrt[5]{y} = 2x$ $\frac{1 - \sqrt[5]{y}}{2} = x$ $f^{-1}(x) = \frac{1 - \sqrt[5]{x}}{2}$
5. Composite function	A **combination** of two or more **functions** to create a new function. $fg(x)$ is the composite function that **substitutes** the function $g(x)$ **into** the function $f(x)$. $fg(x)$ means '**do g first, then f**' $gf(x)$ means '**do f first, then g**'	$f(x) = 5x - 3$, $g(x) = \frac{1}{2}x + 1$ What is $fg(4)$? $g(4) = \frac{1}{2} \times 4 + 1 = 3$ $f(3) = 5 \times 3 - 3 = 12 = fg(4)$ What is $fg(x)$? $fg(x) = 5\left(\frac{1}{2}x + 1\right) - 3 = \frac{5}{2}x + 2$