



Maths



Year 7 Knowledge Organisers



YEAR 7 — ALGEBRAIC THINKING... Sequences



What do I need to be able to do?

By the end of this unit you should be able to:

- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

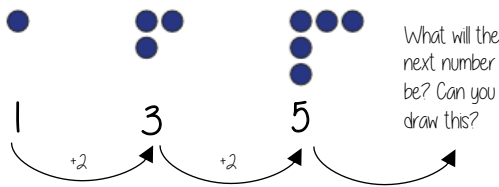
Keywords

- Sequence:** items or numbers put in a pre-decided order
- Term:** a single number or variable
- Position:** the place something is located
- Rule:** instructions that relate two variables
- Linear:** the difference between terms increases or decreases by the same value each time
- Non-linear:** the difference between terms increases or decreases in different amounts
- Difference:** the gap between two terms
- Arithmetic:** a sequence where the difference between the terms is constant
- Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

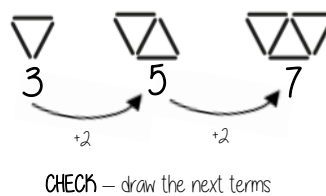


Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



Predict and check terms



Predictions:

Look at your pattern and consider how it will increase.

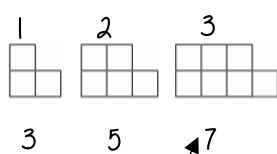
e.g How many lines in pattern 6?

Prediction - 13

If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence

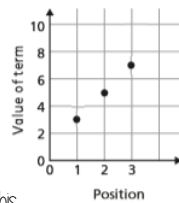


Term: the number or variable (the number of squares in each image)

Position	1	2	3
Term	3	5	7



Graphically



Because the terms increase by the same addition each time this is **linear** - as seen in the graph

Linear and Non Linear Sequences

Linear Sequences - increase by addition or subtraction and the same amount each time

Non-linear Sequences - do not increase by a constant amount - quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence - look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Continue Linear Sequences



7, 11, 15, 19...

How do I know this is a linear sequence?

It increases by adding 4 to each term

How many terms do I need to make this conclusion?

At least 4 terms - two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence

Continue non-linear Sequences



1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?

It increases by multiplying the previous term by 2 - this is a geometric sequence because the constant is multiply by 2

How many terms do I need to make this conclusion?

At least 4 terms - two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence

Explain term-to-term rule

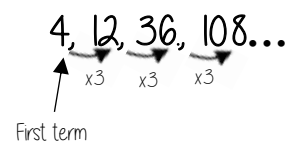
How you get from term to term

Try to explain this in full sentences not just with mathematical notation

Use key maths language - doubles, halves, multiply by two, add four to the previous term etc.

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4



YEAR 7 — ALGEBRAIC THINKING...

Algebraic notation



What do I need to be able to do?

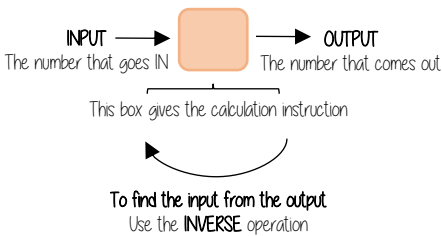
By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions.
- Represent functions graphically.

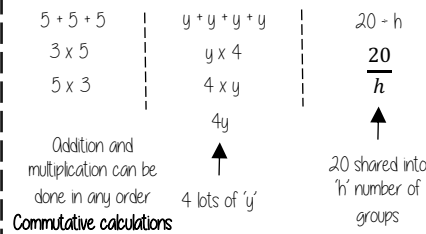
Keywords

- Function:** a relationship that instructs how to get from an input to an output
- Input:** the number/ symbol put into a function
- Output:** the number/ expression that comes out of a function
- Operation:** a mathematical process
- Inverse:** the operation that undoes what was done by the previous operation (The opposite operation)
- Commutative:** the order of the operations do not matter.
- Substitute:** replace one variable with a number or new variable
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Evaluate:** work out
- Linear:** the difference between terms increases or decreases by the same value each time
- Sequence:** items or numbers put in a pre-decided order

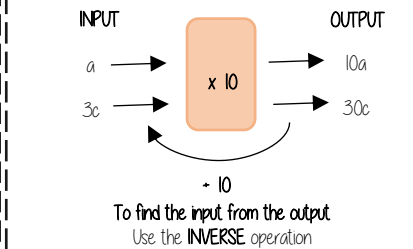
Single function machines



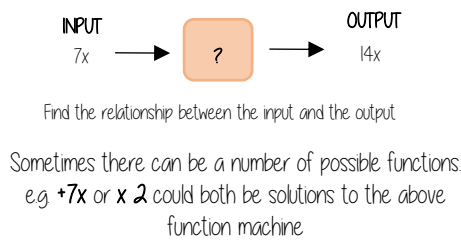
Using letters to represent numbers



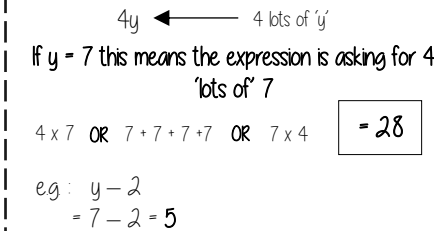
Single function machines (algebra)



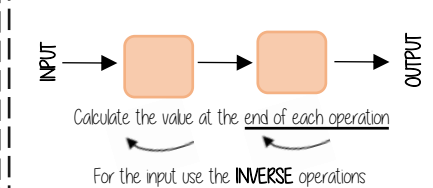
Find functions from expressions



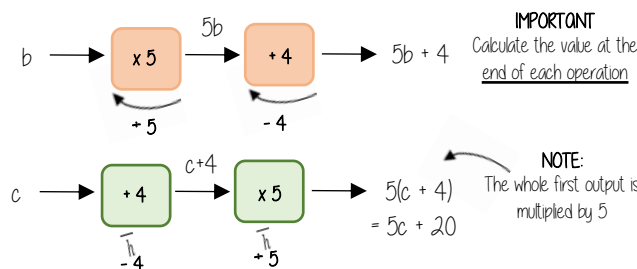
Substitution into expressions



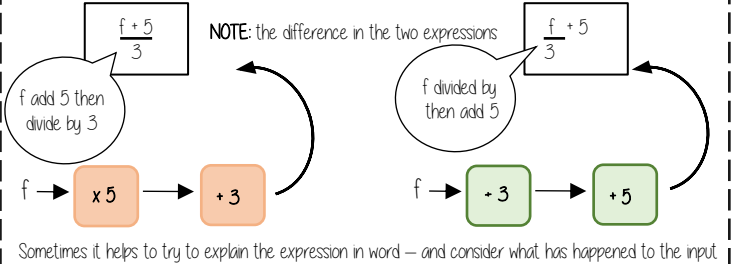
Two step function machines



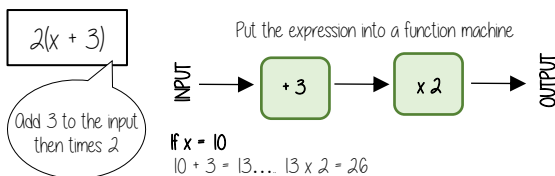
Two step function machines (algebra)



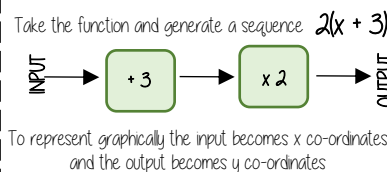
Find functions from expressions



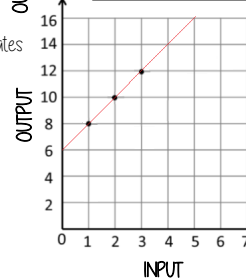
Substitution into an expression



Representing functions graphically

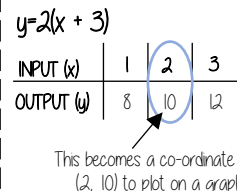
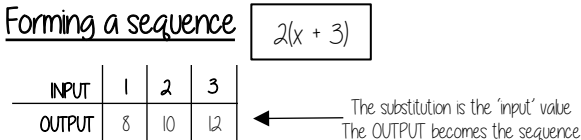


Not all graphs will be linear only those with an integer value for x. Powers and fractions generate differently shaped graphs



NOTE: Because this is a linear graph you can predict other values

Forming a sequence





YEAR 7 — ALGEBRAIC THINKING

Equality and Equivalence

What do I need to be able to do?

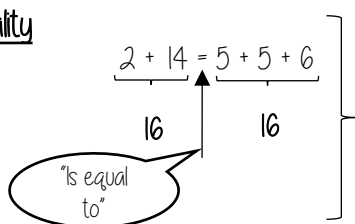
By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

- Equality:** two expressions that have the same value
- Equation:** a mathematical statement that two things are equal
- Equals:** represented by '=' symbol — means the same
- Solution:** the set or value that satisfies the equation
- Solve:** to find the solution
- Inverse:** the operation that undoes what was done by the previous operation (The opposite operation)
- Term:** a single number or variable
- Like:** variables that are the same are 'like'
- Coefficient:** a multiplicative factor in front of a variable e.g. $5x$ (5 is the coefficient, x is the variable)
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equality

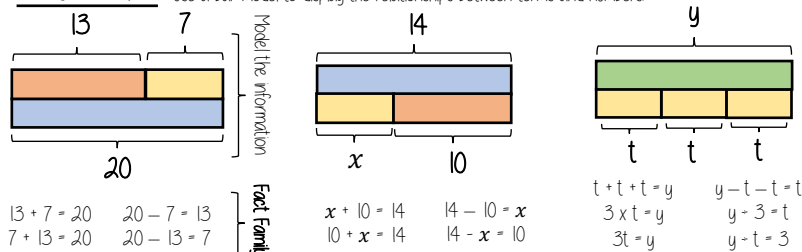


Saying it out loud sometimes helps you to understand equality

The sum on the left has the same result as the sum on the right

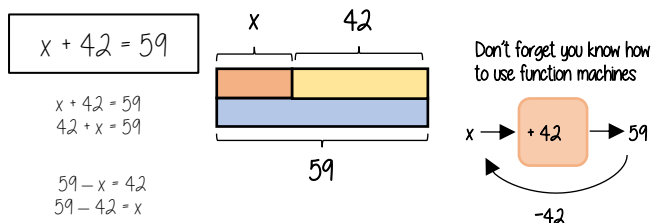
Fact Families

Use a bar model to display the relationships between terms and numbers.

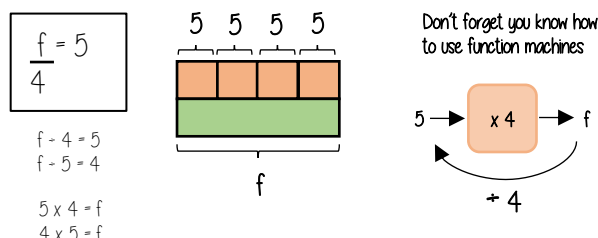


Solve one step equations (+/-)

There is more to this than just spotting the answer

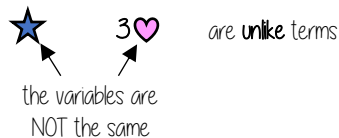
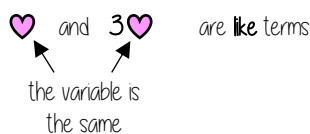


Solve one step equations (x/+)

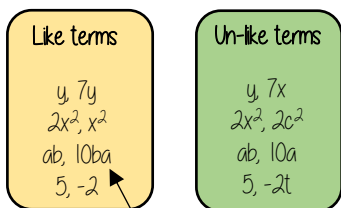


Like and unlike terms

Like terms are those whose variables are the same



Examples and non-examples



Note here ab and ba are commutative operations, so are still like terms

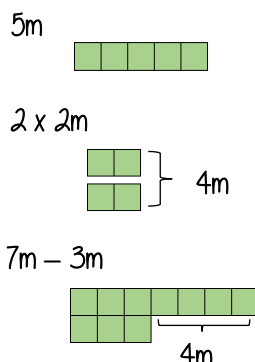
Equivalence

Check equivalence by substitution
e.g. $m = 10$

$5m$	$2 \times 2m$	$7m - 3m$
5×10	$2 \times (2 \times 10)$	$(7 \times 10) - (3 \times 10)$
$= 50$	$= 2 \times 20$	$= 70 - 30$
	$= 40$	$= 40$

Equivalent expressions

Repeat this with various values for m to check

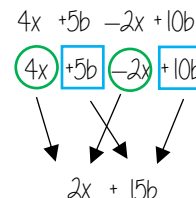


Collecting like terms \equiv symbol

The \equiv symbol means equivalent to
It is used to identify equivalent expressions

Collecting like terms

Only like terms can be combined



Common misconceptions

$$2x + 3x^2 + 4x \equiv 6x + 3x^2$$

Although they both have the x variable x^2 and x terms are unlike terms so can not be collected

YEAR 7 — PLACE VALUE AND PROPORTION



Ordering integers and decimals

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand place value and the number system including decimals
- Understand and use place value for decimals, integers and measures of any size
- Order number and use a number line for positive and negative integers, fractions and decimals;
- use the symbols $=$, \neq , \leq , \geq
- Work with terminating decimals and their corresponding fractions
- Round numbers to an appropriate accuracy
- Describe, interpret and compare data distributions using the median and range

Keywords

- Approximate:** To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with
- Integer:** a whole number that is positive or negative
- Interval:** between two points or values
- Median:** A measure of central tendency (middle, average) found by putting all the data values in order and finding the middle value of the list
- Negative:** Any number less than zero, written with a minus sign
- Place holder:** We use 0 as a place holder to show that there are none of a particular place in a number
- Place value:** The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right
- Range:** The difference between the largest and smallest numbers in a set
- Significant figure:** A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point

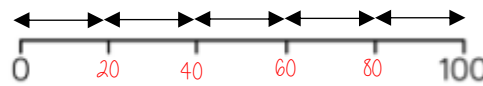
Integer Place Value

Billions			Millions			Thousands			Ones			
H	T	O	H	T	O	H	T	O	H	T	O	
			3	1	4	8	0	3	3	0	2	9

Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine
 1 billion 1,000,000,000
 1 million 1,000,000

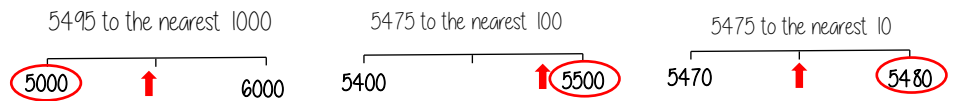
Intervals on a number line



Divide the difference by the number of intervals (gaps).
 Eg $100 \div 5 = 20$

Rounding to the nearest power of ten

If the number is halfway between we "round up"



Compare integers using $<$, $>$, $=$, \neq

- $<$ less than: Two and a half million \leq 2 500 000
- $>$ greater than: 300 000 000 $=$ Three billion
- $=$ equal to: 300 000 000 $=$ Three billion
- \neq not equal to: Six thousand and eighty $<$ 68 000

Range

Spread of the values

Difference between the biggest and smallest
 3 9 8 12
 Range: Biggest value - Smallest value
 $12 - 3 = 9$
 Range = 9

Median

The middle value

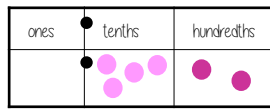
Example 1 Median: put the in order 3 4 8 9 12
 4 3 9 8 12 find the middle number 3 4 8 9 12

Example 2 Median: put the in order 150 154 148 137 148 150 154 158 160
 137 160 158 There are 2 middle numbers Find the midpoint
 152

Decimals

We say "nought point five two"

Five tenths and two hundredths



$$0 \text{ ones, } 5 \text{ tenths and } 2 \text{ hundredths}$$

$$0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$$

$$= 0 + 0.5 + 0.02$$

$$= 0.52$$

Comparing decimals

Which the largest of 0.3 and 0.23?

Ones	Tenths	Hundredths
	0.1, 0.1, 0.1	
	0.1	
	0.1	0.01, 0.01
	0.1	0.01

$$0.3 > 0.23$$

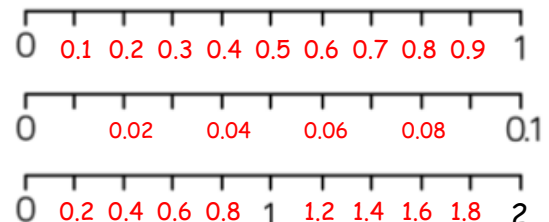
"There are more counters in the furthest column to the left"

0.30
0.23

Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

Decimal intervals on a number line

One whole split into 10 parts makes tenths = 0.1
 One tenth split into 10 parts makes hundredths = 0.01



Round to 1 significant figure

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 37 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

YEAR 7 — PLACE VALUE AND PROPORTION... FDP equivalence



What do I need to be able to do?

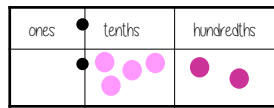
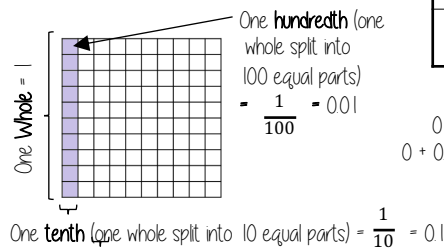
By the end of this unit you should be able to:

- Convert fluently between fractions, decimals & percentages

Keywords

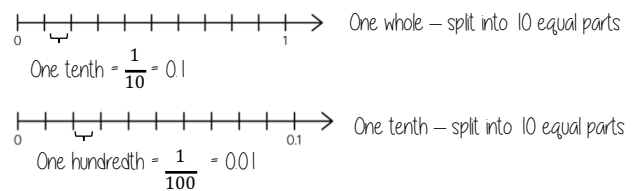
- Fraction:** how many parts of a whole we have
- Decimal:** a number with a decimal point used to separate ones, tenths, hundredths etc.
- Percentage:** a proportion of a whole represented as a number between 0 and 100
- Place value:** the numerical value that a digit has decided by its position in the number
- Placeholder:** a number that occupies a position to give value
- Interval:** a range between two numbers
- Tenth:** one whole split into 10 equal parts
- Hundredth:** one whole split into 100 equal parts
- Sector:** a part of a circle between two radius (often referred to as looking like a piece of pie)
- Recurring:** a decimal that repeats in a given pattern

Tenths and hundredths

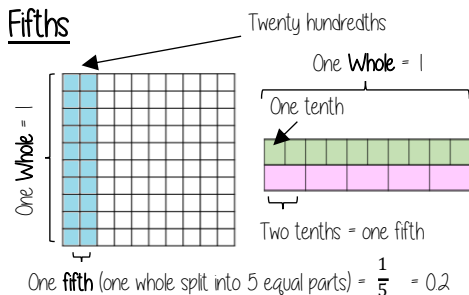


0 ones, 5 tenths and 2 hundredths
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$
 $= 0 + 0.5 + 0.02$
 $= 0.52$

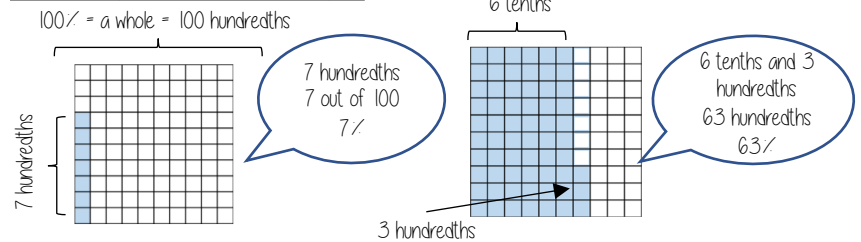
On a number line



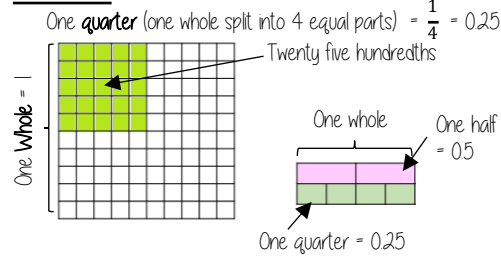
Fifths



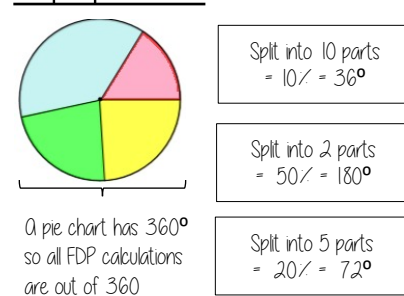
Percentages on a hundred grid



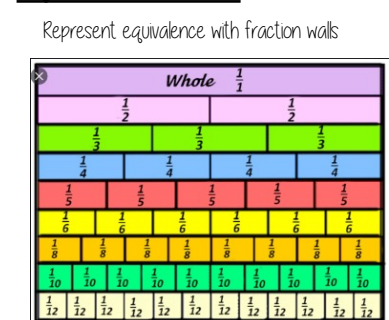
Quarters



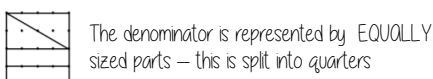
Simple pie charts



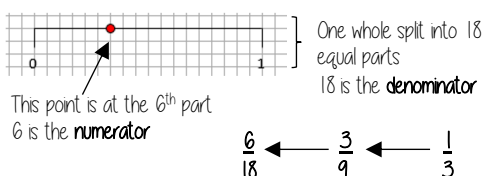
Equivalent fractions



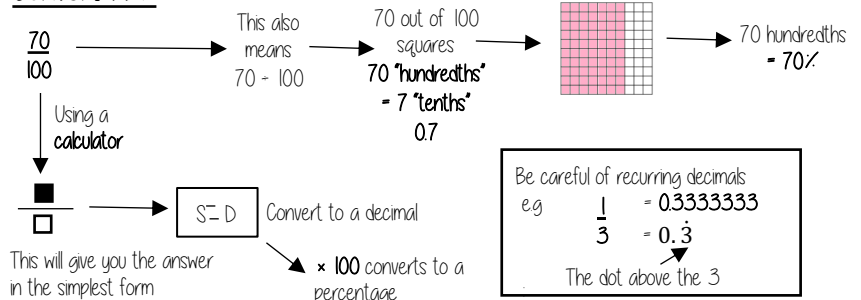
Fractions — on a diagram



Fractions — on a number line



Convert FDP



Be careful of recurring decimals
 e.g. $\frac{1}{3} = 0.3333333$
 $\frac{1}{3} = 0.\dot{3}$
 The dot above the 3

YEAR 7 — APPLICATION OF NUMBER



Solving problems with addition and subtraction

What do I need to be able to do?

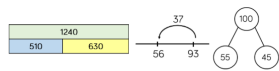
By the end of this unit you should be able to:

- Understand properties of addition/ subtraction
- Use mental strategies for addition/subtraction
- Use formal methods of addition/Subtraction for integers
- Use formal methods of addition/Subtraction for decimals
- Solve problems in context of perimeter
- Solve problems with finance, tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts

Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Inverse:** the operation that undoes what was done by the previous operation (The opposite operation)
- Placeholder:** a number that occupies a position to give value
- Perimeter:** the distance/ length around a 2D object
- Polygon:** a 2D shape made with straight lines
- Balance:** in financial questions — the amount of money in a bank account
- Credit:** money that goes into a bank account
- Debit:** money that leaves a bank account

Addition/ Subtraction with integers



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
-	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

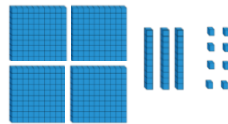
Addition/ Subtraction with decimals

4	.	3	8	
7	.	9	0	+

0 can be used to fill empty places with value

The decimal place acts as the placeholder and aligns the other values

$$5.43 + \frac{8}{10}$$

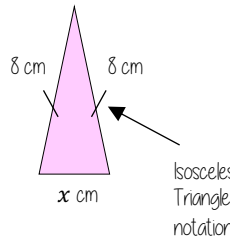


If represents 1 instead of 100

Revisit Fraction — Decimal equivalence
 $5.43 + 0.8$

Solve problems with perimeter

Perimeter is the length around the outside of a polygon



Isosceles Triangle notation

The triangle has a perimeter of 25cm. Find the length of x

$$\begin{aligned} 8\text{cm} + 8\text{cm} + x\text{cm} &= 25\text{cm} \\ 16\text{cm} + x\text{cm} &= 25\text{cm} \\ x\text{cm} &= 9\text{cm} \end{aligned}$$

Solve problems with finance

$$\text{Profit} = \text{Income} - \text{Costs}$$

Credit — Money coming into an account

Debit — Money leaving an account

Money uses a two decimal place system
14.2 on a calculator represents £14.20

Check the units of currency — work in the same unit

Tables and timetables

Distance tables

London	Cardiff	Glasgow	Belfast
211	493	177	
556			
518	392		

This shows the distance between Glasgow and London. It is where their row and column intersect

Bus/ Train timetables

Harton	1005	1045	1130
Bridge	1024	1106	1147
Aville	1051	1133	1205
Ware	1117	1202	1233

Each column represents a journey, each row represents the time the 'bus' arrives at that location

TIME CALCULATIONS — use a number line

Two-way tables

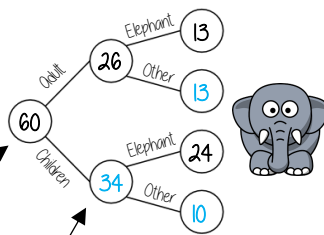
	H	T
H	HH	HT
T	TH	TT

Where rows and columns intersect is the outcome of that action

Frequency trees

60 people visited the zoo one Saturday morning.
26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

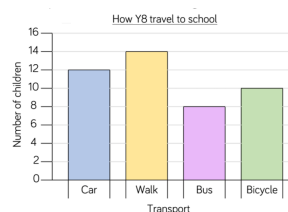
The overall total "60 people"



A frequency tree is made up from part-whole models. One piece of information leads to another

Probabilities or statements can be taken from the completed trees
e.g. 34 children visited the zoo

Bar and line charts



Use addition/ subtraction methods to extract information from bar charts

e.g. Difference between the number of students who walked and took the bus
Walk frequency — bus frequency

When describing changes or making predictions

- Extract information from your data source
- Make comparisons of difference or sum of values
- Put into the context of the scenario

YEAR 7 — APPLICATION OF NUMBER



Solving problems with multiplication and division

What do I need to be able to do?

- By the end of this unit you should be able to:
- Understand and use factors
 - Understand and use multiples
 - Multiply/ Divide integers and decimals by powers of 10
 - Use formal methods to multiply
 - Use formal methods to divide
 - Understand and use order of operations
 - Solve area problems
 - Solve problems using the mean

Keywords

- Array:** an arrangement of items to represent concepts in rows or columns
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Milli: prefix meaning one thousandth
Centi: prefix meaning one hundredth
Kilo: prefix meaning multiply by 1000
Quotient: the result of a division
Dividend: the number being divided
Divisor: the number we divide by

Factors

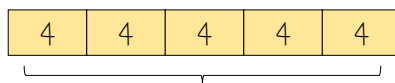
●●●●● Arrays can help represent factors ●●●●●
 ●●●●● Factors of 10 10 x 1 or 1 x 10
 5 x 2 or 2 x 5 1, 2, 5, 10
 The number itself is always a factor

Square numbers have an **ODD** number of factors

- Factors of 4: 1, 2, 4
 Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Be strategic - Lay factors out in pairs can help you not to miss any

Multiples



Bar models can represent by something is a multiple. Eg 20 is a multiple of 4

Lowest Common Multiples

LCM of 9 and 12
 9: 9, 18, 27, 36, 45, 54
 12: 12, 24, 36, 48, 60
 The first time their multiples match LCM = 36

Multiply/ Divide by powers of 10

100s 10s 1s × 100
 3 × 100 = 300
 1s 1/10s 1/100s × 100
 0.03 × 100 = 3

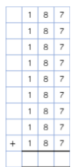
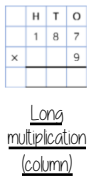
Repeated multiplication and division by powers of 10 is commutative
 ÷ 10 then ÷ 10 → ÷ 100

Metric conversions

Useful Conversions



Multiplication methods



Less effective method especially for bigger multiplication

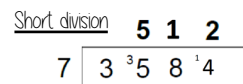
Multiplication with decimals

Perform multiplications as integers
 eg $0.2 \times 0.3 \rightarrow 2 \times 3$
 Make adjustments to your answer to match the question:
 $0.2 \times 10 = 2$
 $0.3 \times 10 = 3$
 Therefore $6 \div 100 = 0.06$

Estimations: Using estimations allows a 'check' if your answer is reasonable

Division methods

$3584 \div 7 = 512$

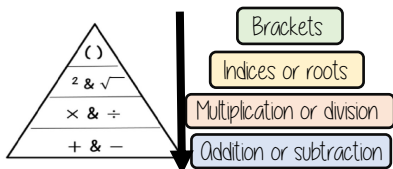


Complex division
 $\div 24 = \div 6 \div 4$
 Break up the divisor using factors

Division with decimals

The placeholder in division methods is essential - the decimal lines up on the dividend and the quotient
 $24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$
 All give the same solution as represent the same proportion
 Multiply the values in proportion until the divisor becomes an integer

Order of operations



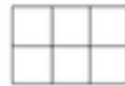
If you have multiple operations from the same tier work from left to right

eg $10 - 3 + 5 \rightarrow 10 - 3 \rightarrow 7 + 5$

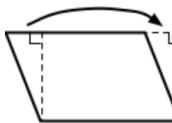
$6 \times 4 + 8 \times 2$
 $24 + 16 = 40$

Area problems

Rectangle
 Base x Perpendicular height



Parallelogram/ Rhombus
 Base x Perpendicular height



Triangle
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$

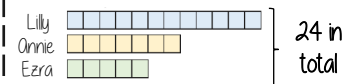


A triangle is half the size of the rectangle it would fit in

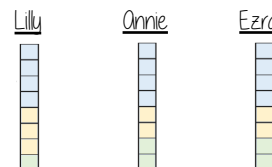
Mean problems

Mean - a measure of average
 It gives an idea of the central value

Lilly, Annie and Ezra have the following cubes



Finding the mean amount is the average amount each person would have if shared out equally



The mean number of blocks would be 8 each

YEAR 7 — APPLICATION OF NUMBER



Fractions and percentages of amounts

What do I need to be able to do?

By the end of this unit you should be able to:

- Find a fraction of a given amount
- Use a given fraction to find the whole or other fractions
- Find the percentage of an amount using mental methods
- Find the percentage of a given amount using a calculator

Keywords

Fraction: how many parts of a whole we have

Equivalent: of equal value

Whole: a number with no fractional or decimal part

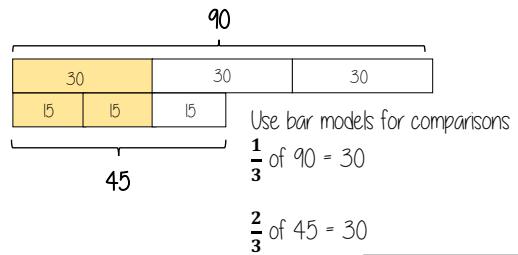
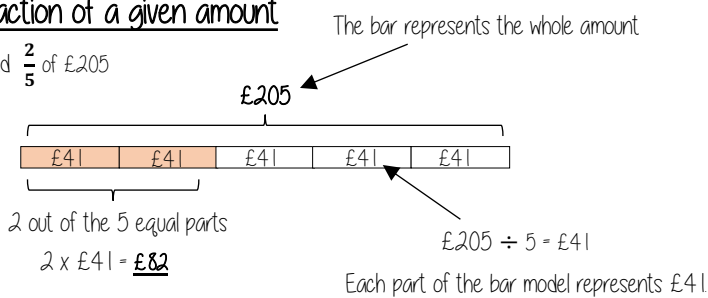
Percentage: parts per 100 (uses the % symbol)

Place Value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Convert: change into an equivalent representation, often fraction to decimal to a percentage cycle.

Fraction of a given amount

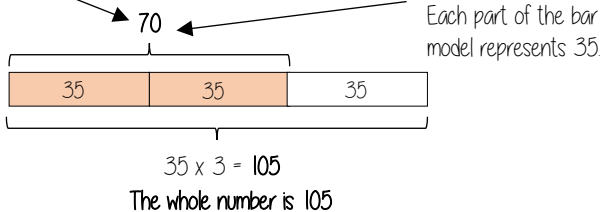
Find $\frac{2}{5}$ of £205



$$\therefore \frac{1}{3} \text{ of } 90 = \frac{2}{3} \text{ of } 45$$

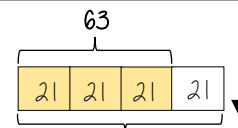
Use a fraction of amount

$\frac{2}{3}$ of a value is 70. What is the whole number?



The wording of the question is important to setting up the bar model

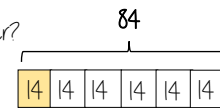
$\frac{3}{4}$ of a number is 63.



Find the whole

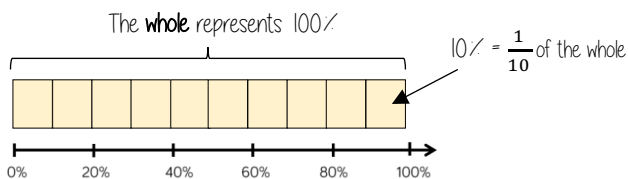
What is $\frac{1}{6}$ of the number?

$$= 14$$



Use the whole to find a given part

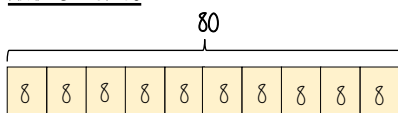
Find the percentage of an amount (Mental methods)



$$10\% = \frac{1}{10} \text{ of the whole} \quad 50\% = \frac{5}{10} = \frac{1}{2} \text{ of the whole}$$

$$20\% = \frac{2}{10} = \frac{1}{5} \text{ of the whole} \quad 5\% = \frac{1}{20} \text{ of the whole}$$

Find 65% of 80



Method 1

$$65\% = 10\% \times 6 + 5\% \\ = (8 \times 6) + 4 \\ = 52$$

Method 2

$$65\% = 50\% + 10\% + 5\% \\ = 40 + 8 + 4 \\ = 52$$

For bigger percentages it is sometimes easier to take away from 100%

Find the percentage of an amount (Calculator methods)



Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$$65\% = \frac{65}{100} = 0.65 \quad \leftarrow \text{The multiplier}$$

$$0.65 \times 80 = \underline{52}$$

Using the percent button

Find 65% of 80

Type 65

Press **SHIFT** **(%)**

Press **80** and then press **=**

This brings up the % button on screen
You will see 65%

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

"of" can represent 'x' in calculator methods

YEAR 7 — DIRECTED NUMBER



Operations with equations and directed numbers

What do I need to be able to do?

By the end of this unit you should be able to:

- Perform calculations that cross zero
- Add/ Subtract directed numbers
- Multiply/ Divide directed numbers
- Evaluate algebraic expressions
- Solve two-step equations
- Use order of operations with directed number

Keywords

Subtract: taking away one number from another.

Negative: a value less than zero.

Commutative: changing the order of the operations does not change the result

Product: multiply terms

Inverse: the opposite function

Square root: a square root of a number is a number when multiplied by itself gives the value (symbol $\sqrt{\quad}$)

Square: a term multiplied by itself.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Perform calculations that cross zero

Number lines are useful to help you visualise the calculation crossing 0

Use the number line to guide subtraction of 6

$4 - 6 = -2$

Start at 4

Find the difference between 6 and -4

From 6 to 0
6
From 0 to -4
4
10 beads between them

Rearrangements of the same equation

$-5 + 5 = 0$

$5 - 5 = 0$

Add directed numbers

$2 + -4 = -2$

Zero pair $(-1 + 1 = 0)$

Two -1 's left $= -2$

$8 + -3 = 5$

Partitioning

$8 + -3 = 5$

$5 + 3 + -3 = 5$

Partition the value to create a zero pair calculation

Generalisation $+ - = -$

Representations $\bullet = -1$
 $\bullet = 1$

Subtract directed numbers

Representation for calculation

$2 - -1 = 3$

Take away one

Start with the representation of 2

$2 - -3 = 5$

Generalisation $- - = +$

Representations $\bullet = -1$
 $\bullet = 1$

"Subtract" - means take away or remove

Multiply/ Divide directed numbers

Two representations of the same calculation

$2 \times -3 = -6$

Negative, Negative calculation

-2×-3

This is the negative of 2×-3

$-2 \times -3 = 6$

The act of making counters into their negative is turning them over

Divisions are the inverse operations

Evaluate algebraic expressions

$a = 5$

$b = -4$

$a^2 = 5^2$

$a^2 = 25$

$b^2 = (-4)^2$

$b^2 = 16$

With negative numbers the brackets are important so that it performs -4×-4 .

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

$3b - 2a = 3(-4) - 2(5) = -12 - 10 = -22$

Two-step equations

Bar Model

$4x + 2 = 10$

$10 - 4x = 2$

Representing the same question (use fact families)

Function machine

$x \rightarrow x4 \rightarrow +2 \rightarrow 10$

Inverse operations to find x

Use order of operations

Brackets

Indices or roots

Multiplication or division

Addition or subtraction

Remember square roots have a positive and negative value

Brackets around negative substitutions helps remove calculation errors

x	-3	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9

YEAR 7 — FRACTIONAL THINKING



Addition and subtraction of fractions

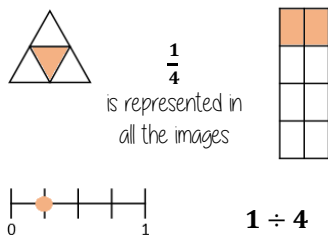
What do I need to be able to do?

- By the end of this unit you should be able to:
- Convert between mixed numbers and fractions
 - Add/Subtract unit fractions (same denominator)
 - Add/Subtract fractions (same denominator)
 - Add/Subtract fractions from integers
 - Use equivalent fractions
 - Add/Subtract any fractions
 - Add/Subtract improper fractions and mixed numbers
 - Use fractions in algebraic contexts

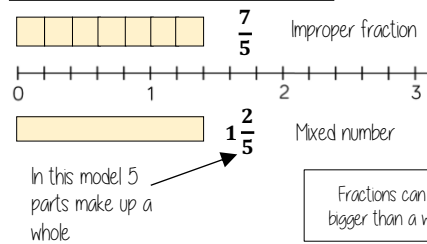
Keywords

- Numerator**: the number above the line on a fraction. The top number. Represents how many parts are taken
- Denominator**: the number below the line on a fraction. The number represent the total number of parts
- Equivalent**: of equal value
- Mixed numbers**: a number with an integer and a proper fraction
- Improper fractions**: a fraction with a bigger numerator than denominator
- Substitute**: replace a variable with a numerical value
- Place value**: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

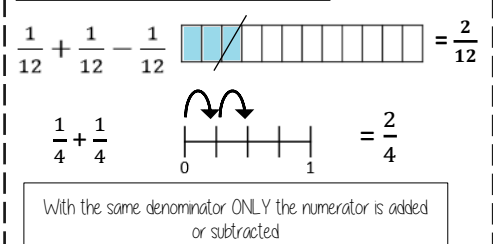
Representing Fractions



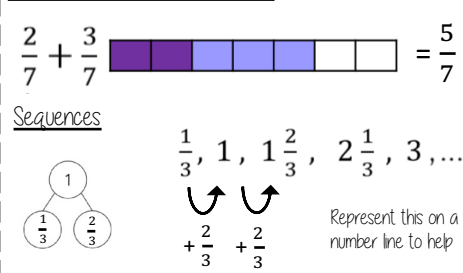
Mixed numbers and fractions



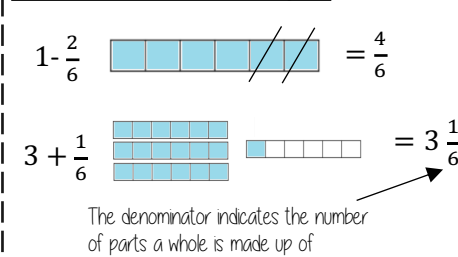
Add/Subtract unit fractions Same denominator



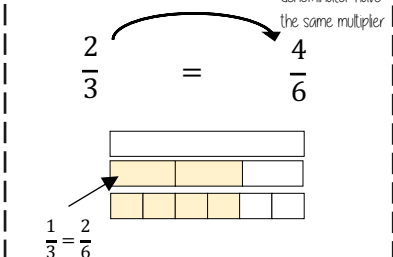
Add/Subtract fractions Same denominator



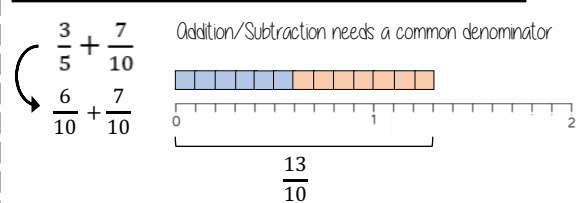
Add/Subtract from integers



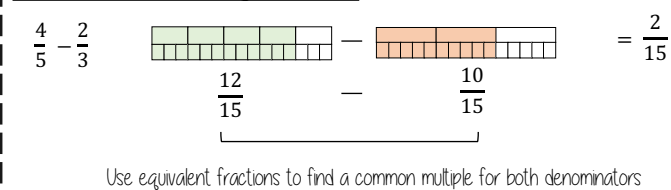
Equivalent fractions



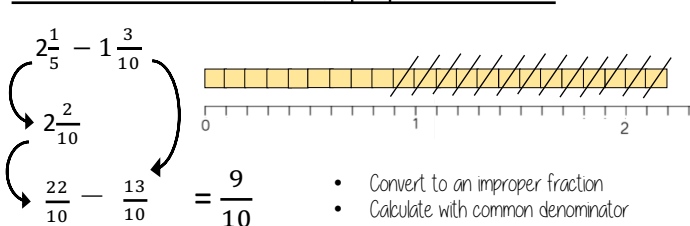
Add/Subtraction fractions (common multiples)



Add/Subtraction any fractions



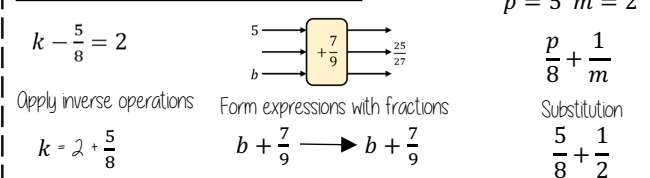
Add/Subtraction fractions (improper and mixed)



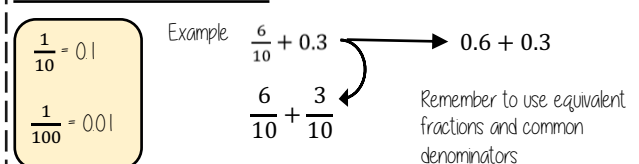
Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Fractions in algebraic contexts



Fractions and decimals



YEAR 7 — LINES AND ANGLES



Constructing, measuring and using geometric notation

What do I need to be able to do?

By the end of this unit you should be able to:

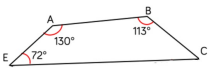
- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

Keywords

- Polygon:** A 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two angles the same size
- Right-angled triangle:** a triangle with a right angle
- Frequency:** the number of times a data value occurs
- Sector:** part of a circle made by two radii touching the centre
- Rotation:** turn in a given direction
- Protractor:** equipment used to measure angles
- Compass:** equipment used to draw arcs and circles

Letter and labelling convention

The letter in the middle is the angle
The arc represents the angle

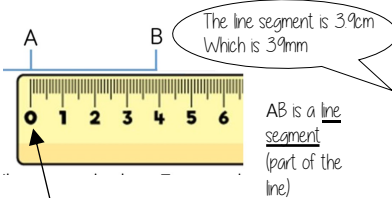


Angle Notation: three letters ABC
This is the angle at B = 113°

Line Notation: two letters EC
The line that joins E to C

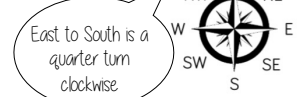
Draw and measure line segments

Conversions: $1\text{cm} = 10\text{mm}$, $1\text{m} = 100\text{cm}$



Make sure the start of the line is at 0.

Angles as measures of turn



East to South is a quarter turn clockwise

Clockwise

Anti-Clockwise

Quarter Turn

90°

Clockwise

Half Turn

180°

Three-quarter Turn

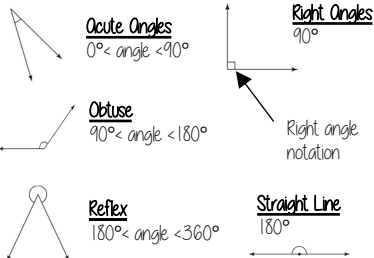
270°

Anti-Clockwise

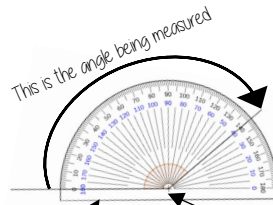
Full Turn

360°

Classify angles



Measure angles to 180°

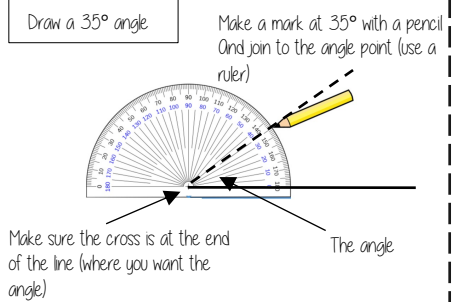


The base line follows the line segment

Make sure the cross is at the point the two lines meet

Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

Draw angles up to 180°



Parallel and Perpendicular lines

Parallel lines
Straight lines that never meet (Have the same gradient)

Perpendicular lines
Straight lines that meet at 90°

Angles over 180°

Use your knowledge of straight lines 180° and angles around a point 360°

360° - smaller angle = reflex angle

Measure the smaller angle first (less than 180°)

Properties of Quadrilaterals

Square
All sides equal size
All angles 90°
Opposite sides are parallel

Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel

Trapezium
One pair of parallel lines

Rhombus
All sides equal size
Opposite angles are equal

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Draw Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$

Use a protractor to draw This is 192°

Polygons

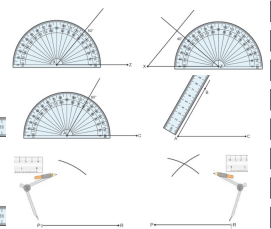
3	- Triangle	5	- Pentagon	8	- Octagon
4	- Quadrilateral	6	- Hexagon	9	- Nonagon
		7	- Heptagon	10	- Decagon

SAS, SSS, ASA constructions

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



If all the sides and angles are the same, it is a **regular polygon**

YEAR 7 — LINES AND ANGLES



Geometric reasoning

What do I need to be able to do?

By the end of this unit you should be able to:

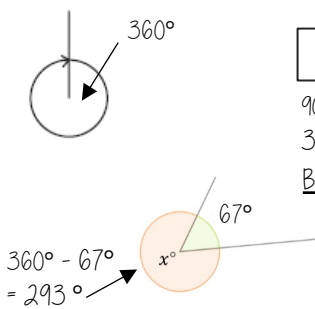
- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

Keywords

- Vertically Opposite:** angles formed when two or more straight lines cross at a point
- Interior Angles:** angles inside the shape
- Sum:** total, add all the interior angles together
- Convex Quadrilateral:** a four-sided polygon where every interior angle is less than 180°
- Concave Quadrilateral:** a four-sided polygon where one interior angle exceeds 180°
- Polygon:** A 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two sides the same size
- Right-angled triangle:** a triangle with a right angle

Sum of angles at a point

The sum of angles around a point is 360°



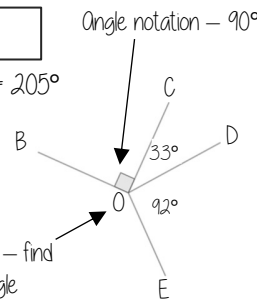
Find angle BOE

$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

$$360^\circ - 205^\circ$$

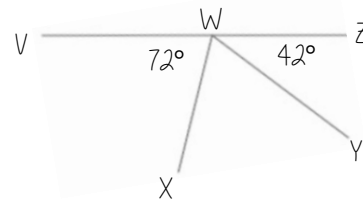
$$\text{BOE} = 155^\circ$$

Angle notation — find this missing angle



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

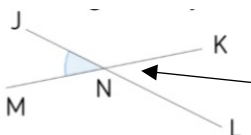


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

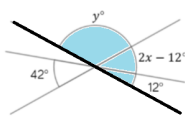


Angle JNM is vertically opposite to angle KNL

$$\text{JNM} = \text{KNL}$$

Vertically opposite angles are the same

Other angle rules still apply. Look for straight line sums and angles around a point.

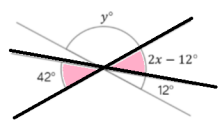


Form equations with information from diagrams:

$$2x - 12 = 42$$

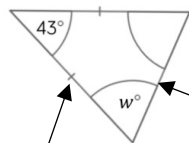
$$2x = 54$$

$$x = 27^\circ$$



Sum of angles in triangles

Sum of interior angles in a triangle = 180°



The two base angles will be the same size

Look at triangle notation. This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

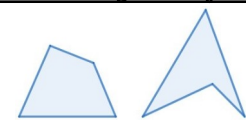
A triangle can only have ONE right angle



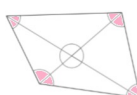
Have a go! Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals

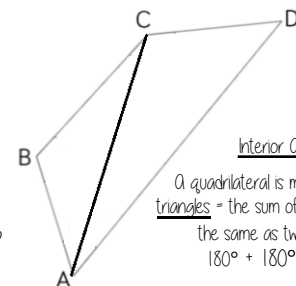
Sum of interior angles in a quadrilateral = 360°



Convex Quadrilateral Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

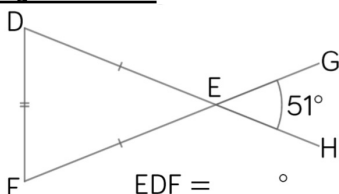


Interior Angles
A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles:
 $180^\circ + 180^\circ = 360^\circ$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation

Keep working out clear and notes together



- Angle DEF = 51° because it is a vertically opposite angle DEF = GEH
- Triangle DEF is isosceles (triangle notation) \therefore EDF = EFD and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$ $129^\circ \div 2 = 64.5^\circ$
- Angle EDF = 64.5°

YEAR 7 — REASONING WITH NUMBER



Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

Dividend: the number being divided

Divisor: the number we divide by

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equation: a mathematical statement that two things are equal

Quotient: the result of a division

Mental methods for addition/ subtraction

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller e.g. $\times 0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{l} 12 \times 3 = 36 \\ \div 10 \downarrow \div 100 \downarrow \div 1000 \downarrow \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \downarrow \quad \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

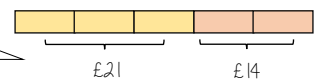
Methods for addition $2.3 + 2.4$

$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

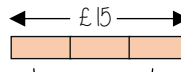
Mental methods for fractions

Use bar models where possible

I've spent $\frac{2}{5}$ of my money I have £21 left



How much did they have to begin with?



$$\frac{3}{3} + \frac{2}{3}$$

What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

The correct estimation would be $200 + 900 = 1100$.

Number facts

Use $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 5$$

For division you must consider the impact of the divisor becoming smaller or bigger.
Smaller — the answer will be bigger (It is being shared into less parts)
Bigger — the answer will be smaller (It is being shared into more parts)

Algebraic facts

$$2a + 2b = 10 \quad \text{Everything } \times 2$$

$$0.1a + 0.1b = 0.5 \quad \text{Everything } \div 10$$

$$a + b = 5$$

$$a + b + 2 = 7 \quad \text{Add 2 to the total}$$

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER



Sets and probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

- Set:** collection of things
- Element:** each item in a set is called an element
- Intersection:** the overlapping part of a Venn diagram (AND \cap)
- Union:** two ellipses that join (OR \cup)
- Mutually Exclusive:** events that do not occur at the same time
- Probability:** likelihood of an event happening
- Bias:** a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice
- Fair:** there is zero bias, and all outcomes have an equal likelihood
- Random:** something happens by chance and is unable to be predicted

Identify and represent sets

The **universal set** has this symbol ξ — this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things — you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$

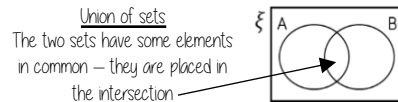
$A = \{1, 4, 9, 16, 25, 36, 49\}$

All the numbers in set A are square number and between 1 and 50

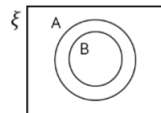
Interpret and create Venn diagrams



Mutually exclusive sets
The two sets have nothing in common
No overlap



Union of sets
The two sets have some elements in common — they are placed in the intersection



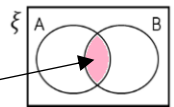
Subset
All of set B is also in Set A so the ellipse fits inside the set

The box

Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

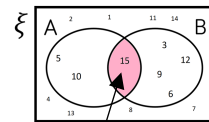
Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$

$A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

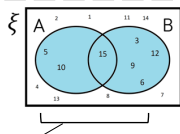


The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B

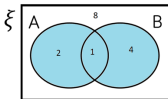


$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

The notation for this is $A \cup B$



This Venn shows the **number of elements** in each set

Sample space — for single events



A sample space for rolling a six-sided dice is $S = \{1, 2, 3, 4, 5, 6\}$



A sample space for this spinner is $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event



Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$$P(\text{Blue}) = \frac{4}{10}$$

← There are 4 blue sectors
← There are 10 sectors overall

$$= \frac{2}{5}$$

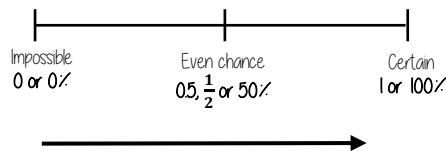
Probability notation
 $P(\text{event})$

Probability can be a fraction, decimal or percentage value

$$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$$

Probability is always a value between 0 and 1

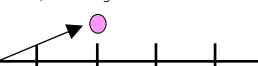
The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of **NOT** getting a blue ball is $\frac{4}{5}$
The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof



What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors.

Conjecture: a statement that might be true (based on reasoning) but is not proven.

Counterexample: a special type of example that disproves a statement.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3×1.5

Not an integer

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10

1, 2, 5, 10

10×1 or 1×10

The number itself is always a factor

Factors and expressions

$x \ x \ x \ x \ x \ x$

$6x \times 1$ OR $6 \times x$

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$x \ x$

$2x \times 3$

$x \ x \ x$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers



odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful — an extra counter is added to each new row



Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)

1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

Counterexamples



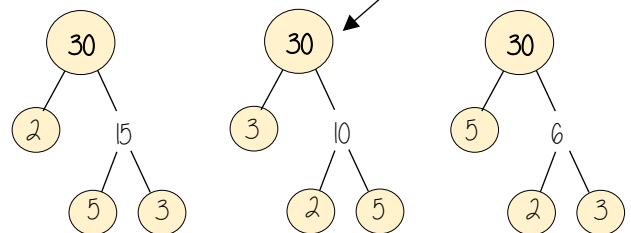
This sequence isn't doubling it is adding 2 each time

A pattern that is noticed for many cases

Only one counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

eg 60: 30×2 or $2 \times 3 \times 5 \times 2$
150: 30×5 or $2 \times 3 \times 5 \times 5$