



# Maths



## Year 8 Knowledge Organisers



# YEAR 8 - PROPORTIONAL REASONING...



## Ratio and Scale

### What do I need to be able to do?

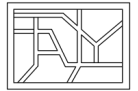
By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved

### Keywords

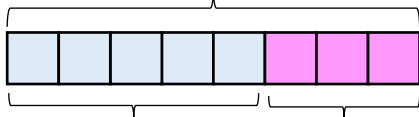
- Ratio:** a statement of how two numbers compare  
**Equal Parts:** all parts in the same proportion, or a whole shared equally  
**Proportion:** a statement that links two ratios  
**Order:** to place a number in a determined sequence  
**Part:** a section of a whole  
**Equivalent:** of equal value  
**Factors:** integers that multiply together to get the original value  
**Scale:** the comparison of something drawn to its actual size



### Representing a ratio

"For every 5 boys there are 3 girls"

This is the "whole" - boys and girls together

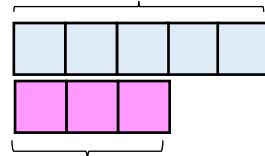


This represents the 5 boys      This represents the 3 girls

5:3

This represents the 5 boys

Double Number Line



This is the "whole" - boys and girls together

This represents the 3 girls

### Order is Important

"For every dog there are 2 cats"



1:2

The ratio has to be written in the same order as the information is given

e.g. 2:1 would represent 2 dogs for every 1 cat. ✗

### Simplifying a ratio

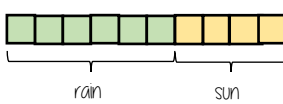
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

+ by 2

3:2



Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiplies into them is 2)

"For every 3 days of rain there are 2 days of sun" - when this happens twice the ratio becomes 6:4.

### Ratio In (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1n

The question states that this part has to be 1 unit. Therefore Divide by 4

4:20  
↓  
1:5

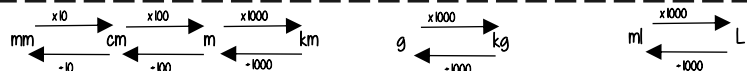
This side has to be divided by 4 too - to keep in proportion

\*n\* the n part does not have to be an integer for this type of question

### Units are important:

When using a ratio - all parts should be in the same units

Useful Conversions



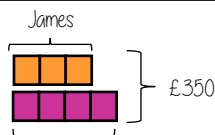
### Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3:4



Lucy

£350 ÷ 7 = £50

□ = one part = £50

Find the value of one part

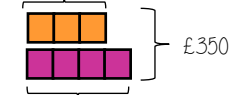
Whole: £350  
7 parts to share between (3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 x £50 = £150

(x50 3:4 x50)  
→ £150:£200



Lucy = 4 x £50 = £200

### Finding a value given 1:n (or n:1)

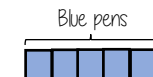
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue: Red

5:1

□ = one part = 10 pens



Red pens

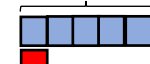
One unit = 10 pens

Put back into the question

Blue: Red

(x10 5:1 x10)  
→ 50:10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

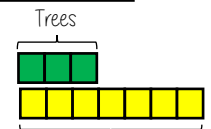
There are 50 Blue Pens

### Ratio as a fraction



Trees: Flowers

3:7



Ratio

There are 3 parts for trees

Flowers

Fraction of trees

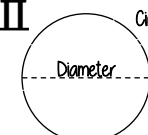
Number of parts in group  
Total number of parts

3  
10

Fraction

Tree parts 3 + Flower parts 7 = 10

Pi II



Circumference

Diameter

The ratio of a circles circumference to its diameter

# YEAR 8 - PROPORTIONAL REASONING...



## Multiplicative Change

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

### Keywords

- Proportion:** a statement that links two ratios
- Variable:** a part that the value can be changed
- Axes:** horizontal and vertical lines that a graph is plotted around
- Approximation:** an estimate for a value
- Scale Factor:** the multiple that increases/ decreases a shape in size
- Currency:** the system of money used in a particular country
- Conversion:** the process of changing one variable to another
- Scale:** the comparison of something drawn to its actual size.

### Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40

12 cans of pop = £7.20

2 cans of pop = £1.20

Sometimes this is easiest if you work out how much one unit is worth first e.g 1 can of pop = £0.60

This multiplier is the same in the same way that this would be for ratio

### Conversion Graphs

Compare two variables

This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare - then find the associated point by using your graph. Using a ruler helps for accuracy. Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

### Conversion between currencies



£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

Currency can be converted using a conversion graph

Convert 630 Rupees into Pounds

630 - 90 = 7

£7 = 630 Rupees

### Ratio between similar shapes

Angles in similar shapes do not change. e.g if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.

3m 8m 4.5m ?m

Corresponding sides

3m : 4.5m

1m : 1.5m

8m : 12m

1m : 1.5m

Note: Simplify to the same ratio

### Understand Scale Factor

The two rectangles are similar.

3m 8m 4.5m ?m

3 x 15 = 4.5

This is a multiplicative change

Use corresponding sides to calculate a scale factor

Scale factor can also be calculated by:

Bigger corresponding side

Smaller corresponding side

Small corresponding side

Big corresponding side

x SF

- SF

### Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life

1cm : 30cm

10cm : 300cm

The car in real life is 210cm

Image : Real life

1cm : 30cm

7cm : 210cm

### Interpret maps with scale factors

mm cm m km

x 10 x 100 x 1000

1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

250 x 100 = 25000

For every 1cm on my map is 25000cm in real life.

# YEAR 8 - PROPORTIONAL REASONING...



## Multiplying and Dividing Fractions

### What do I need to be able to do?

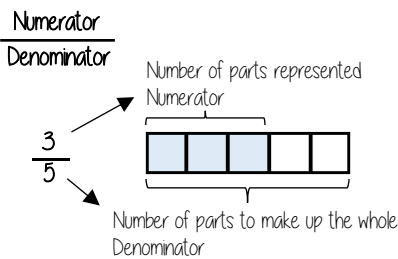
- By the end of this unit you should be able to:
- Carry out any multiplication or division using fractions and integers
  - Solutions can be modelled, described and reasoned

### Keywords

- Numerator**: the number above the line on a fraction. The top number. Represents how many parts are taken  
**Denominator**: the number below the line on a fraction. The number represent the total number of parts.  
**Whole**: a positive number including zero without any decimal or fractional parts.  
**Commutative**: an operation is commutative if changing the order does not change the result  
**Unit Fraction**: a fraction where the numerator is one and denominator a positive integer.  
**Non-unit Fraction**: a fraction where the numerator is larger than one.  
**Dividend**: the amount you want to divide up.  
**Divisor**: the number that divides another number.  
**Quotient**: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient  
**Reciprocal**: a pair of numbers that multiply together to give 1

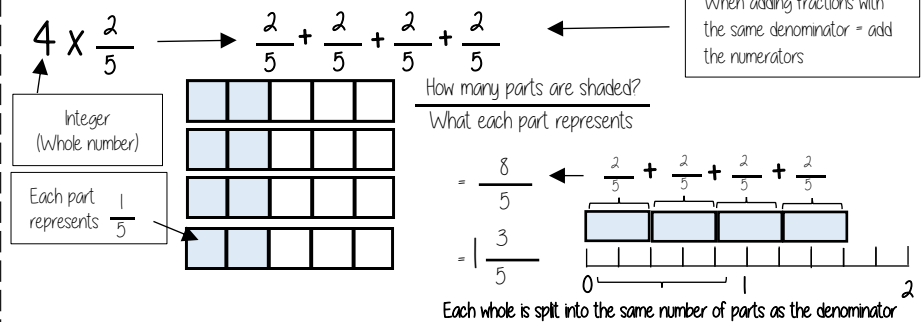


### Representing a fraction

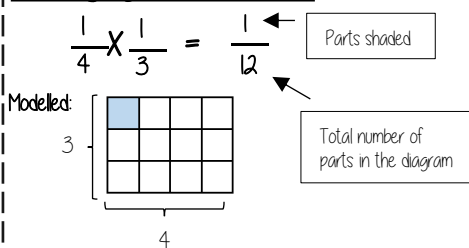


ALL PARTS of a fraction are of equal size

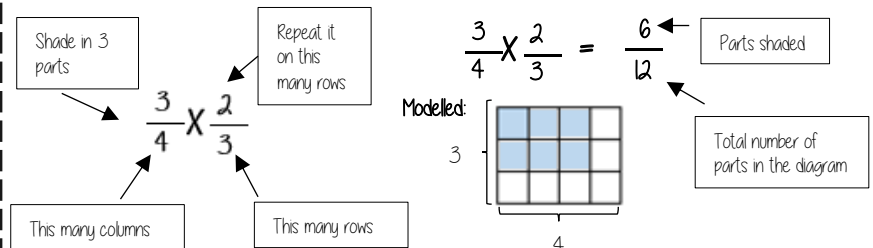
### Repeated addition = multiplication by an integer



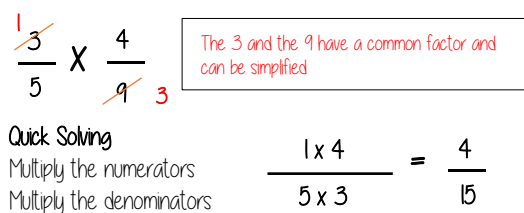
### Multiplying unit fractions



### Multiplying non-unit fractions

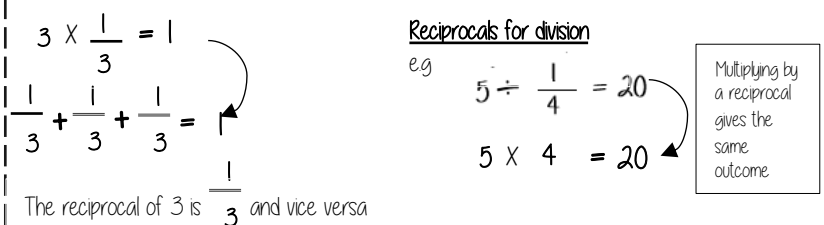


### Quick Multiplying and Cancelling down

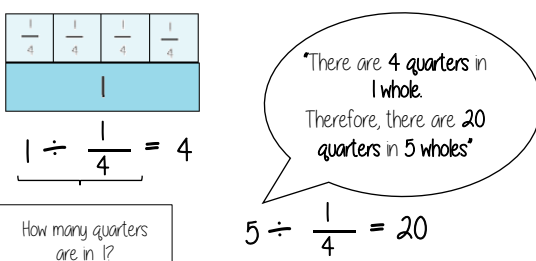


### The reciprocal

When you multiply a number by its reciprocal the answer is always 1

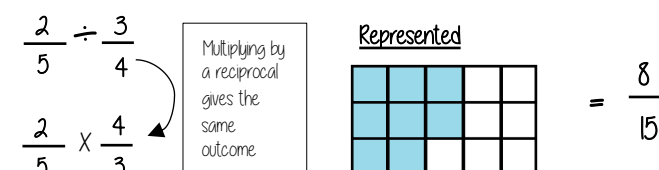


### Dividing an integer by an unit fraction



### Dividing any fractions

Remember to use reciprocals



# YEAR 8 - REPRESENTATIONS...



## Working in the Cartesian plane

### What do I need to be able to do?

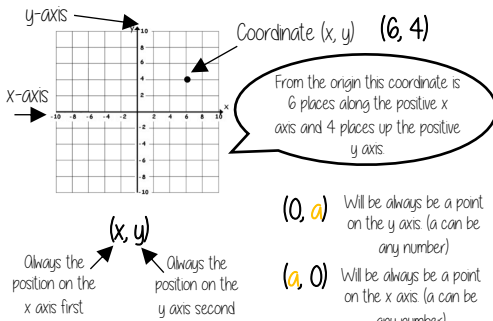
By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot  $y = mx + c$  graphs

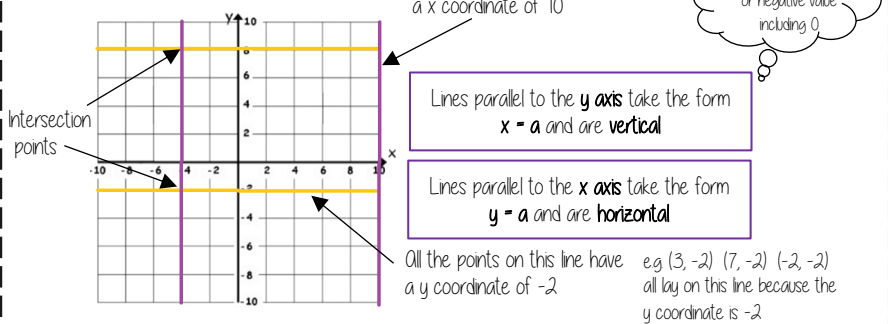
### Keywords

- Quadrant:** four quarters of the coordinate plane.
- Coordinate:** a set of values that show an exact position.
- Horizontal:** a straight line from left to right (parallel to the x axis)
- Vertical:** a straight line from top to bottom (parallel to the y axis)
- Origin:** (0,0) on a graph. The point the two axes cross
- Parallel:** Lines that never meet
- Gradient:** The steepness of a line
- Intercept:** Where lines cross

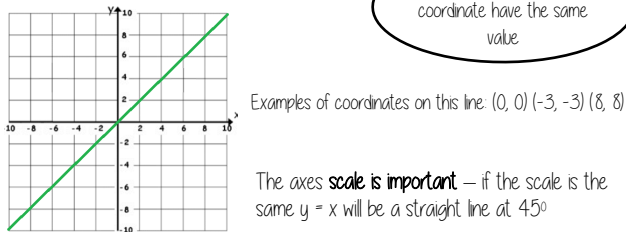
### Coordinates in four quadrants



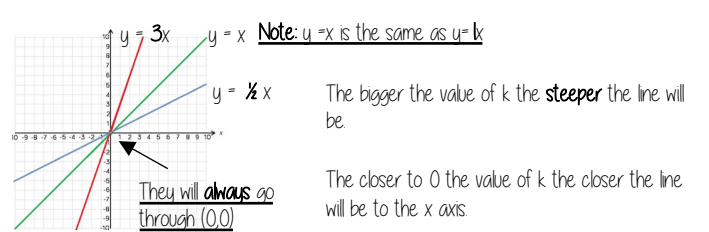
### Lines parallel to the axes



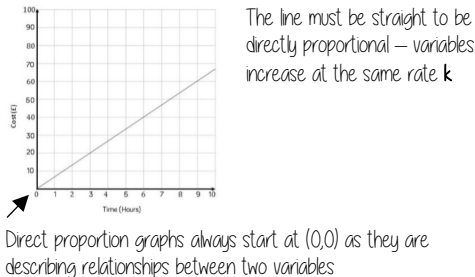
### Recognise and use the line $y=x$



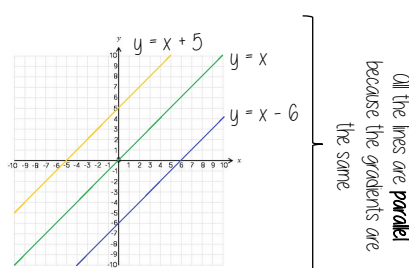
### Recognise and use the lines $y=kx$



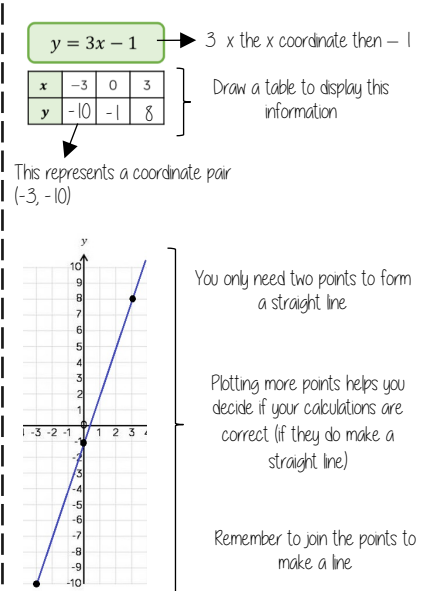
### Direct Proportion using $y=kx$



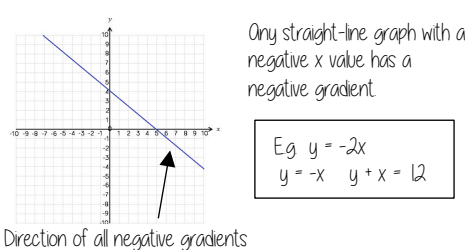
### Lines in the form $y = x + a$



### Plotting $y = mx + c$ graphs



### Lines with negative gradients



# YEAR 8 - REPRESENTATIONS...



# Representing Data

## What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and interpret scatter graphs
- Describe correlation and relationships
- Identify different types of non-linear relationships
- Design and complete an ungrouped frequency table
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables

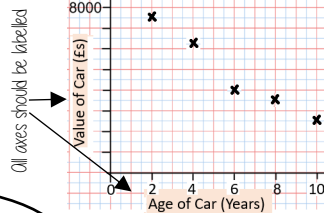
## Keywords

- Variable:** a quantity that may change within the context of the problem
- Relationship:** the link between two variables (items) E.g Between sunny days and ice cream sales
- Correlation:** the mathematical definition for the type of relationship.
- Origin:** where two axes meet on a graph
- Line of best fit:** a straight line on a graph that represents the data on a scatter graph
- Outlier:** a point that lies outside the trend of graph
- Quantitative:** numerical data
- Qualitative:** descriptive information, colours, genders, names, emotions etc.
- Continuous:** quantitative data that has an infinite number of possible values within its range.
- Discrete:** quantitative or qualitative data that only takes certain values
- Frequency:** the number of times a particular data value occurs.

## Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

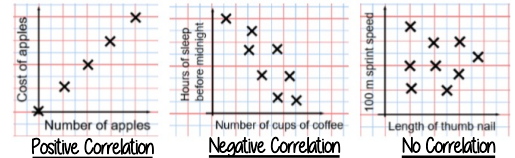
- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

## Linear Correlation



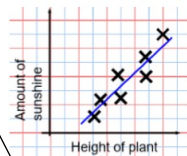
- Positive Correlation:** As one variable increases so does the other variable
- Negative Correlation:** As one variable increases the other variable decreases
- No Correlation:** There is no relationship between the two variables

## The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

### Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



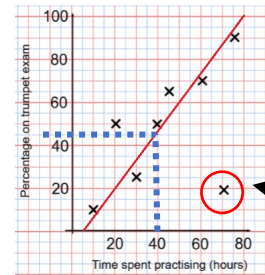
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

## Using a line of best fit

**Interpolation** is using the line of best fit to estimate values inside our data point

e.g 40 hours revising predicts a percentage of 45



**Extrapolation** is where we use our line of best fit to predict information outside of our data

\*\*This is not always useful – in this example you cannot score more than 100% So revising for longer can not be estimated\*\*

This point is an "outlier" It is an outlier because it doesn't fit this model and stands apart from the data

## Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were  
3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

Number of siblings	Frequency
0	2
1	3
2	4
3	2
4	1

2 people had 0 siblings. This means there are 0 siblings to be counted here

0  
3  
2 + 2 + 2 + 2 OR 2 x 4 = 8  
3 + 3 OR 3 x 2 = 6

Best represented by discrete data (Not always a number)

2 people have 3 siblings so there are 6 siblings in total

**OVERALL there are**  
0 + 3 + 8 + 6 + 4  
Siblings = 21 siblings

## Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

**Discrete Data**  
The groups do not overlap

Cost of TV (£)	Tally	Frequency
101 - 150	THL II	7
151 - 200	THL THL I	11
201 - 250	THL	5
251 - 300	III	3

We do not know the exact value of each item in a group – so an estimate would be used to calculate the overall total (Midpoint)

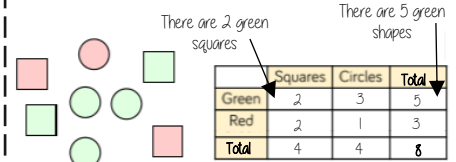
**Continuous Data**  
To make sure all values are included inequalities represent the subgroups

x	Frequency
Weight(g) 40 < x ≤ 50	1
50 < x ≤ 60	3
60 < x ≤ 70	5

e.g this group includes every weight bigger than 60kg, up to and including 70kg

## Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



### Using your two-way table

To find a fraction  
e.g What fraction of the items are red? 3 red items  
but 8 items in total =  $\frac{3}{8}$

**Interchanging:** Use your fraction, decimal percentage equivalence knowledge

# YEAR 8 - REPRESENTATIONS...



# Tables and Probability

## What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram
- Systematically list outcomes
- Find the probability from two-way tables
- Find the probability from Venn diagrams

## Keywords

**Outcomes:** the result of an event that depends on probability

**Probability:** the chance that something will happen

**Set:** a collection of objects

**Chance:** the likelihood of a particular outcome

**Event:** the outcome of a probability – a set of possible outcomes

**Biased:** a built in error that makes all values wrong by a certain amount

**Union:** Notation 'U' meaning the set made by comparing the elements of two sets

## Construct sample space diagrams



Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

This is the set notation to list the outcomes S =

In between the { } are a; the possible outcomes

$$S = \{ 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T \}$$

## Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

This is the set notation that represents the question P

What is the probability that an outcome has an even number and a tails?

$$P(\text{Even number and Tails}) = \frac{3}{12}$$

In between the ( ) is the event asked for

There are three even numbers with tails

Numerator: the event

Denominator: the total number of outcomes

There are twelve possible outcomes

## Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

$$P(\text{Girl walk to school}) = \frac{21}{100}$$

The total number of items

The event

The total in the set

## Product Rule

The number of items in event a

x

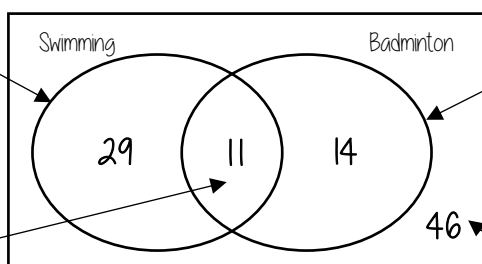
The number of items in event b

## Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club  
40 went swimming, 25 went to badminton and 11 went to both

This whole curve includes everyone that went swimming  
Because 11 did both we calculate just swimming by 40 - 11

The intersection represents both Swimming AND badminton



This whole curve includes everyone that went to badminton  
Because 11 did both we calculate just badminton by 25 - 11

The number outside represents those that did neither badminton or swimming

$$P(\text{Just swimming}) = \frac{29}{100}$$

$$100 - 29 - 11 - 14$$

# YEAR 8 - ALGEBRAIC TECHNIQUES...



## Brackets, Equations & Inequalities

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

### Keywords

- Simplify:** grouping and combining similar terms
- Substitute:** replace a variable with a numerical value
- Equivalent:** something of equal value
- Coefficient:** a number used to multiply a variable
- Product:** multiply terms
- Highest Common Factor (HCF):** the biggest factor (or number that multiplies to give a term)
- Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

### Form expressions

For unknown variables, a letter is normally used in its place

More than - **ADD**

Less than/ difference - **SUBTRACT**

e.g. 4 more than t  $\longrightarrow$   $t + 4$   
 8 less than k  $\longrightarrow$   $k - 8$

Only similar terms can be grouped together

e.g. Find the perimeter of this shape  
 (Perimeter = length around outside of shape)

$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

### Directed numbers

$++ \longrightarrow +$   
 $-- \longrightarrow +$   
 $+ - \longrightarrow -$   
 $- + \longrightarrow -$

e.g.  $a = -5$  and  $b = 2$   
 $a^2 = a \times a = -5 \times -5 = 25$   
 $b + a = 2 + -5 = -3$

### Multiply single brackets

$3(2x + 4)$

Different representations of  $3(2x + 4) = 6x + 12$

### Factorise into a single bracket

$8x + 4$



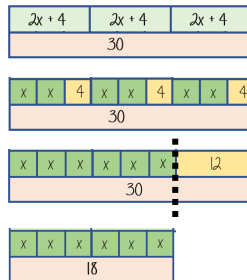
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:  
 $8x + 4 \equiv 2(4x + 2)$   
 This is factorised but the HCF has not been used

### Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$-6 \quad -6$

Substitute to check your answer. This could be negative or a fraction or decimal

$\begin{matrix} x \\ 3 \end{matrix} \quad x = 3$

### Simple Inequalities

- < less than
- ≤ Less than or equal to
- > More than
- ≥ More than or equal to

$x < 10$   
 Say this out loud  
 "x is a value less than 10"

$10 > x$   
 Say this out loud  
 "10 is more than the value"

Note:  
 $x < 10$  and  $10 > x$  represent the same values

$x + 2 \leq 20$   
 "my value + 2 is less than or equal to 20"

$x \leq 18$   
 The biggest the value can be is 18

### Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \longrightarrow x \times 3 \longrightarrow +2 \longrightarrow 11$

$3x + 2 > 11$

Solve

$x \longleftarrow -3 \longleftarrow -2 \longleftarrow 11$

$x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11 \checkmark$        $10 \times 3 + 2 = 32 \checkmark$

### Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes  $\equiv$

Formula

A rule written with all mathematical symbols  
 e.g. area of a rectangle  $A = b \times h$

# YEAR 8 - ALGEBRAIC TECHNIQUES...



# Sequences

## What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the  $n$ th term
- Recognise geometric sequences and other sequences that arise

## Keywords

**Sequence:** items or numbers put in a pre-decided order

**Term:** a single number or variable

**Position:** the place something is located

**Linear:** the difference between terms increases or decreases (+ or -) by a constant value each time

**Non-linear:** the difference between terms increases or decreases in different amounts, or by  $\times$  or  $\div$

**Difference:** the gap between two terms

**Arithmetic:** a sequence where the difference between the terms is constant

**Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

## Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

**Fibonacci Sequence** – look out for this type of sequence

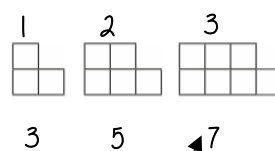
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



## Sequence in a table and graphically

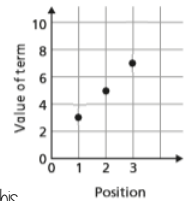
**Position:** the place in the sequence



"The term in position 3 has 7 squares"

**Term:** the number or variable (the number of squares in each image)

**Graphically**



**In a table**

Position	1	2	3
Term	3	5	7

+2 +2

Because the terms increase by the same addition each time this is **linear** – as seen in the graph

## Sequences from algebraic rules

This is substitution!

$$3n + 7$$

$$3n^2 + 7$$

This will be linear – note the single power of  $n$ . The values increase at a constant rate

This is not linear as there is a power for  $n$

$$2n - 5$$

Substitute the number of the term you are looking for in place of 'n'

eg  
 $1^{\text{st}}$  term =  $2(1) - 5 = -3$   
 $2^{\text{nd}}$  term =  $2(2) - 5 = -1$   
 $100^{\text{th}}$  term =  $2(100) - 5 = 195$

## Checking for a term in a sequence

Form an equation

Is 201 in the sequence  $3n - 4$ ?

$$3n - 4 = 201$$

Term to check

Algebraic rule

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

## Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

$$(2n)^2$$

2 times whatever  $n$  squared is

2 times  $n$  then square the answer

eg  
 $1^{\text{st}}$  term =  $2 \times 1^2 = 2$   
 $2^{\text{nd}}$  term =  $2 \times 2^2 = 8$   
 $100^{\text{th}}$  term =  $2 \times 100^2 = 20000$

eg  
 $1^{\text{st}}$  term =  $(2 \times 1)^2 = 4$   
 $2^{\text{nd}}$  term =  $(2 \times 2)^2 = 16$   
 $100^{\text{th}}$  term =  $(2 \times 100)^2 = 40000$

$$n(n + 5)$$

eg  
 $1^{\text{st}}$  term =  $1(1 + 5) = 6$   
 $2^{\text{nd}}$  term =  $2(2 + 5) = 14$   
 $100^{\text{th}}$  term =  $100(100 + 5) = 10500$

You don't need to expand the expression

## H Finding the algebraic rule

This is the 4 times table  $\rightarrow$  4, 8, 12, 16, 20....

$$4n$$

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

# YEAR 8 - ALGEBRAIC TECHNIQUES...



## Indices

### What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

### Keywords

**Base:** The number that gets multiplied by a power

**Power:** The exponent – or the number that tells you how many times to use the number in multiplication

**Exponent:** The power – or the number that tells you how many times to use the number in multiplication

**Indices:** The power or the exponent

**Coefficient:** The number used to multiply a variable

**Simplify:** To reduce a power to its lowest term

**Product:** Multiply

### Addition/ Subtraction with indices

$5x^2 + 4x^4$   
 Coefficient      Power  
 $\underbrace{5}_{\text{Term}} x^{\underbrace{2}_{\text{Term}}} + \underbrace{4}_{\text{Term}} x^{\underbrace{4}_{\text{Term}}}$   
 Expression

Each square represents  $x^2$  and each cube represents  $x^4$

Only similar terms can be simplified  
If they have different powers, they are unlike terms

$5x^2 + 2x^2 \rightarrow$ 
 $\rightarrow 7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4 \rightarrow$ 
 $\rightarrow 2x^2 + 7x^4$

### Multiply expressions with indices

$4b \times 3a$   
 $\equiv 4 \times b \times 3 \times a$   
 $\equiv 4 \times 3 \times b \times a$   
 $\equiv 12 ab$

$5t \times 9t$   
 $\equiv 5 \times t \times 9 \times t$   
 $\equiv 5 \times 9 \times t \times t$   
 $\equiv 45 t^2$

$2b^4 \times 3b^2$   
 $\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$   
 $\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$   
 $\equiv 6 b^6$

There are often misconceptions with this calculation but break down the powers

### Addition/ Subtraction laws for indices

$3^5 \times 3^2 \rightarrow 3^7$   
 $= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

### Divide expressions with indices

$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$

$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b}} \rightarrow \frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23 a^7 y^2}{5 d b^6}$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

# YEAR 8 - DEVELOPING NUMBER...

## Fractions & Percentages



### What do I need to be able to do?

- By the end of this unit you should be able to:
- Convert between FDP less than and more than 100.
  - Increase or decrease using multipliers.
  - Express an amount as a percentage.
  - Find percentage change.

### Keywords

- Percent:** parts per 100 – written using the % symbol  
**Decimal:** a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.  
**Fraction:** a fraction represents how many parts of a whole value you have.  
**Equivalent:** of equal value.  
**Reduce:** to make smaller in value.  
**Growth:** to increase/ to grow.  
**Integer:** whole number, can be positive, negative or zero.  
**Invest:** use money with the goal of it increasing in value over time (usually in a bank).

### Convert FDP



70/100 → This also means 70 - 100 → 70 out of 100 squares → 70 'hundredths' = 7 'tenths' = 0.7 → 70 hundredths = 70%.

Using a calculator:  $\frac{70}{100} = 0.7$  → Convert to a decimal → × 100 converts to a percentage.

Be careful of recurring decimals  
 e.g.  $\frac{1}{3} = 0.333333$   
 $\frac{1}{3} = 0.\dot{3}$   
 The dot above the 3

### Fraction/ Percentage of amount



Find  $\frac{3}{5}$  of £60 → £60 → £12, £12, £12, £12, £12 → £36

Remember  $\frac{3}{5} = 60\%$   
 10% of £60 = £6  
 50% of £60 = £30  
 60% of £60 = £36

Remember  $\frac{3}{5} = 60\% = 0.6$   
 60% of £60 = 0.6 × 60 = £36

### Convert FDP < and > 100%

100 hundredths = 10 tenths = 100%  
 40 hundredths = 4 tenths = 40%  
 140 hundredths = 14 tenths = 140%

$100\% + 40\% = 1 + 0.4 = 1.40$

### Percentage decrease: Multipliers

100% → 42% → Decrease by 58% → Multiplier Less than 1

$100\% - 58\% = 42\%$   
 $100 - 0.58 = 0.42$

### Percentage increase: Multipliers

100% → 12% → Increase by 12% → Multiplier More than 1

$100\% + 12\% = 112\%$   
 $100 + 0.12 = 1.12$

### Express as a % - Non-calculator

7 per every 10 are orange →  $\frac{7}{10}$  → This means that 70 per every 100 are orange →  $\frac{70}{100}$  → 70%

27 per every 50 shaded →  $\frac{27}{50}$  → 54 per every 100 shaded →  $\frac{54}{100}$  → 54%

Denominator 100      Equivalent fractions

### Express as a % - Calculator

Rosie  $\frac{13}{30}$  →  $\frac{13}{30}$  → × 100 → 43.3333...% → 43%

This the same as 13 ÷ 30

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage.

### Percentage change

I bought a phone for £200. A year later sold it for £125.

All values of change compare to the ORIGINAL value.

Percentage loss:  $\frac{75}{200} \times 100 = 37.5\%$

I bought a house for £180,000, I later sold it for £216,000.

Percentage profit:  $\frac{36000}{180000} \times 100 = 20\%$

Difference in value / Original value × 100

### Choose appropriate method

The language and wording of the question is the key.

Have you represented the question in a bar model?  
 Can you use a calculator?

# YEAR 8 - DEVELOPING NUMBER...



# Standard Form

## What do I need to be able to do?

- By the end of this unit you should be able to:
- Write numbers in standard form and as ordinary numbers
  - Order numbers in standard form
  - Add/ Subtract with standard form
  - Multiply/ Divide with standard form
  - Use a calculator with standard form

## Keywords

- Standard (index) Form:** A system of writing very big or very small numbers
- Commutative:** an operation is commutative if changing the order does not change the result
- Base:** The number that gets multiplied by a power
- Power:** The exponent – or the number that tells you how many times to use the number in multiplication
- Exponent:** The power – or the number that tells you how many times to use the number in multiplication
- Indices:** The power or the exponent
- Negative:** A value below zero.

## Positive powers of 10

1 billion – 1 000 000 000  
 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices  $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices  $10^a \div 10^b = 10^{a-b}$

## Numbers between 0 and 1

0.054  
 $= 5.4 \times 10^{-2}$

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$10^0$	•	$10^{-1}$	$10^{-2}$	$10^{-3}$
0	•	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

## Standard form with numbers > 1

Any number between 1 and less than 10  $\rightarrow A \times 10^n$  ← Any integer

### Example

$3.2 \times 10^4$   
 $= 3.2 \times 10 \times 10 \times 10 \times 10$   
 $= 32000$

### Non-example

$(0.8) \times 10^4$   
 $5.3 \times 10^{(0.7)}$

## Negative powers of 10

0.001

10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$10^1$	$10^0$	•	$10^{-1}$	$10^{-2}$	$10^{-3}$
0	0	•	0	0	1

$1 \times \frac{1}{1000}$   
 $1 \times 10^{-3}$

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

## Order numbers in standard form

$10^{-2}$	$10^1$	$10^0$	•	$10^{-1}$	$10^{-2}$	$10^{-3}$	$10^{-4}$
$6.4 \times 10^{-2}$	$2.4 \times 10^2$	$3.3 \times 10^0$		$1.3 \times 10^{-1}$			
0.064	240	1		0.13			

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

## Mental calculations

$6.4 \times 10^2 \times 1000$  Not in Standard Form  
 $= 6.4 \times 10^2 \times 10^3$   
 $= 6.4 \times 10^5$  Use addition for indices rule

$(2 \times 10^3) \div 4$  Divide the values  
 $= (2 \div 4) \times 10^3$   
 $= 0.5 \times 10^3$

$(8) \times 10^5 \times (3)$  Not in Standard Form  
 $= 24 \times 10^5$   
 $= 2.4 \times 10^1 \times 10^5$  Use addition for indices rule  
 $= 2.4 \times 10^6$

### Remember the layout for standard form

Any number between 1 and less than 10  $\rightarrow A \times 10^n$  ← Any integer

## Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end

### Method 1

$6 \times 10^5 + 8 \times 10^5$   
 $= 600000 + 800000$   
 $= 1400000$   
 $= 1.4 \times 10^6$

### Method 2

$= (6 + 8) \times 10^5$   
 $= 14 \times 10^5$   
 $= 1.4 \times 10^1 \times 10^5$   
 $= 1.4 \times 10^6$

This is not the final answer

More robust method  
 Less room for misconceptions  
 Easier to do calculations with negative indices  
 Can use for different powers

Only works if the powers are the same

## Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$  Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$(15 \div 0.3) \times 10^5 \div 10^3$   
 $= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices  
 $a^m \times a^n = a^{m+n}$

Subtraction law for indices  
 $a^m \div a^n = a^{m-n}$

## Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press  $\times 10^4$  Then press 5 (for the power)  
 Press  $\times$   
 Input 3.9 and press  $\times 10^3$  Then press 3 (for the power)  
 Press  $=$

This gives you the solution



Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press **SHIFT** **SETUP** and then press 7 for sci mode  
 Choose a degree of accuracy so in most cases press 2

Answer:  $5.5 \times 10^8$

# YEAR 8 - DEVELOPING NUMBER...



## Number Sense

### What do I need to be able to do?

#### to do?

- By the end of this unit you should be able to:
- Round numbers to powers of 10 and 1 sf
  - Round numbers to any dp
  - Estimate solutions
  - Calculate using order of operations
  - Calculate with money, units of measurement and time

### Keywords

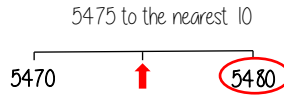
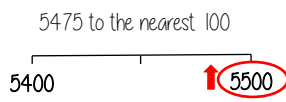
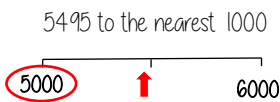
- Significant:** Place value of importance  
**Round:** Making a number simpler but keeping its value close to what it was  
**Decimal:** Place holders after the decimal point  
**Overestimate:** Rounding up — gives a solution higher than the actual value  
**Underestimate:** Rounding down — gives a solution lower than the actual value  
**Metric:** A system of measurement  
**Balance:** The amount of money in a bank account  
**Deposit:** Putting money into a bank account

### Round to powers of 10 and 1 sig. figure

**R** If the number is halfway between we "round up"

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

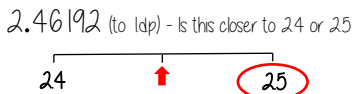


### Round to decimal places

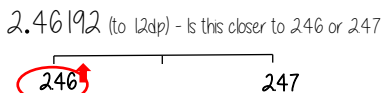
2.46192

Focus on the numbers after the decimal point

"To 1dp" — to one number after the decimal  
 "To 2dp" — to two numbers after the decimal



2.4 6192 This shows the number is closer to 2.5



2.46 192 This shows the number is closer to 2.46

### Estimate the calculation

Round to 1 significant figure to estimate

$$4.2 + 6.7 \approx 4 + 7 \approx 11$$

This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$$21.4 \times 3.1 \approx 20 \times 3 \approx 60$$

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths — it helps you identify calculation errors.

### Order of operations

**R**

**Brackets** Operations in brackets are calculated first

**Other** operations e.g powers, roots,

**Multiplication/ Division**

They are carried out in the order from left to right in the question

**Addition/ Subtraction**

They are carried out in the order from left to right in the question

### Calculations with money

**Debit** - You have £0 or more in an account

**Credit** - You have less than £0 in an account

Money calculations are to 2dp

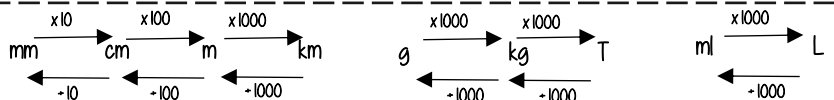


Using a calculator — ensure you are working in the correct units  
 $£130 + 50p = 130 + 50$  (in pence)  
 $= 130 + 0.50$  (in pounds)

£1 = 100p



### Units are important: Useful Conversions



### Metric measures of length

Kilo = 1000 x meter      Centi =  $\frac{1}{100}$  x meter

Milli =  $\frac{1}{1000}$  x meter

### Time and the calendar



**12 Months** — one year = 52 weeks

31 days — Jan, March, May, July, Aug, Oct, Dec

30 days — April, June, Sept, Nov

28 days — Feb (29 leap year)

**1 week** — 7 days

Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

**1 day** — 24 hours

**1 hour** — 60 minutes

**1 minute** — 60 seconds

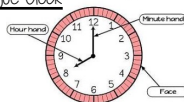
Use a number line for time calculations!

### Units of weight/ capacity

Weight = g, kg, t

Capacity (volume of liquid) = ml, L

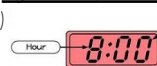
#### Analogue Clock



#### 12-hour clock

- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

#### Digital Clock (24-hour times)



#### 24-hour clock

- 0-11 (morning hours)
- 12-23 (afternoon hours)

# YEAR 8 - DEVELOPING GEOMETRY...



# Angles in parallel lines and polygons

## What do I need to be able

### to do?

By the end of this unit you should be able to:

- Identify alternate angles
- Identify corresponding angles
- Identify co-interior angles
- Find the sum of interior angles in polygons
- Find the sum of exterior angles in polygons
- Find interior angles in regular polygons

## Keywords

**Parallel:** Straight lines that never meet

**Angle:** The figure formed by two straight lines meeting (measured in degrees)

**Transversal:** A line that cuts across two or more other (normally parallel) lines

**Isosceles:** Two equal size lines and equal size angles (in a triangle or trapezium)

**Polygon:** A 2D shape made with straight lines

**Sum:** Addition (total of all the interior angles added together)

**Regular polygon:** All the sides have equal length; all the interior angles have equal size.

## Basic angle rules and notation



**Acute Angles**  
 $0^\circ < \text{angle} < 90^\circ$

**Right Angles**  
 $90^\circ$

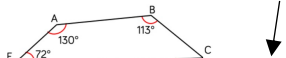
**Obtuse**  
 $90^\circ < \text{angle} < 180^\circ$

Right angle notation

**Reflex**  
 $180^\circ < \text{angle} < 360^\circ$

**Straight Line**  
 $180^\circ$

The letter in the middle is the angle  
The arc represents the part of the angle



**Angle Notation:** three letters ABC

This is the angle at B =  $113^\circ$

**Line Notation:** two letters EC

The line that joins E to C.

**Vertically opposite angles**

Equal

**Angles around a point**

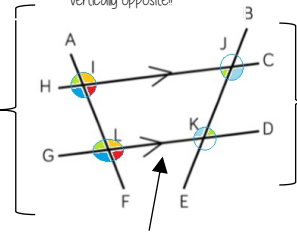
$360^\circ$

## Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

Lines OF and BE are transversals (lines that bisect the parallel lines)

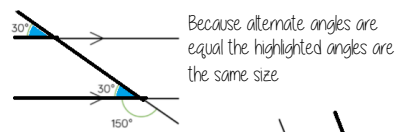
Corresponding angles often identified by their "F shape" in position



Alternate angles often identified by their "Z shape" in position

This notation identifies parallel lines

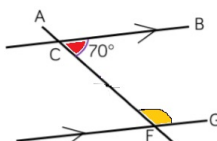
## Alternate/ Corresponding angles



Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size

## Co-interior angles



Because co-interior angles have a sum of  $180^\circ$  the highlighted angle is  $110^\circ$

As angles on a line add up to  $180^\circ$  co-interior angles can also be calculated from applying alternate/ corresponding rules first

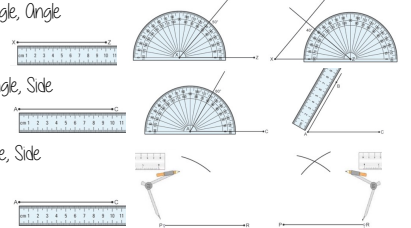
## Triangles & Quadrilaterals

Link to steps

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



## Properties of Quadrilaterals

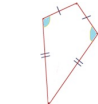
**Square**  
All sides equal size  
All angles  $90^\circ$   
Opposite sides are parallel



**Rectangle**  
All angles  $90^\circ$   
Opposite sides are parallel



**Rhombus**  
All sides equal size  
Opposite angles are equal



**Parallelogram**

Opposite sides are parallel  
Opposite angles are equal  
Co-interior angles

**Trapezium**

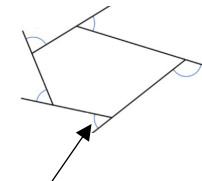
One pair of parallel lines

**Kite**

No parallel lines  
Equal lengths on top sides  
Equal lengths on bottom sides  
One pair of equal angles

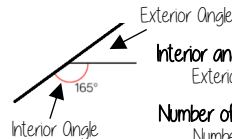
## Sum of exterior angles

Exterior angles all add up to  $360^\circ$



**Exterior Angles**  
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles



Interior angle + Exterior angle = straight line =  $180^\circ$   
Exterior angle =  $180 - 165 = 15^\circ$

Number of sides =  $360^\circ \div \text{exterior angle}$   
Number of sides =  $360 \div 15 = 24$  sides

## Sum of interior angles

**Interior Angles**

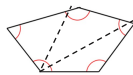
The angles enclosed by the polygon



This is an **irregular** polygon - the sides and angles are different sizes

$$(\text{number of sides} - 2) \times 180$$

$$\text{Sum of the interior angles} = (5 - 2) \times 180$$

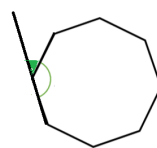


This shape can be made from three triangles  
Each triangle has  $180^\circ$

$$\text{Sum of the interior angles} = 3 \times 180 = 540^\circ$$

Remember this is **all** of the interior angles added together

## Missing angles in regular polygons



$$\text{Exterior angle} = 360 \div 8 = 45^\circ$$

$$\text{Interior angle} = \frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135^\circ$$

Exterior angles in regular polygons =  $360^\circ \div \text{number of sides}$

Interior angles in regular polygons =  $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

# YEAR 8 - DEVELOPING GEOMETRY...



## Area of trapezia and Circles

### What do I need to be able to do?

#### to do?

By the end of this unit you should be able to:

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of a circle
- Find the area of compound shapes
- Find the perimeter of compound shapes

### Keywords

**Congruent:** The same

**Area:** Space inside a 2D object

**Perimeter:** Length around the outside of a 2D object

**Pi ( $\pi$ ):** The ratio of a circle's circumference to its diameter.

**Perpendicular:** At an angle of  $90^\circ$  to a given surface

**Formula:** A mathematical relationship/ rule given in symbols. Eg  $b \times h = \text{area of rectangle/ square}$

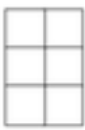
**Infinity ( $\infty$ ):** A number without a given ending (too great to count to the end of the number) – never ends

**Sector:** A part of the circle enclosed by two radii and an arc.

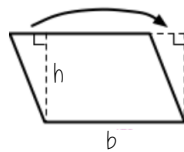
### Area – rectangles, triangles, parallelograms



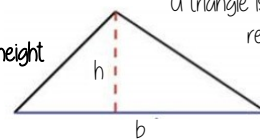
Rectangle  
Base x Height



Parallelogram/ Rhombus  
Base x Perpendicular height



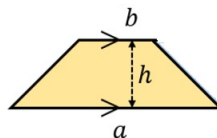
Triangle  
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



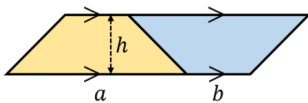
A triangle is half the size of the rectangle it would fit in

### Area of a trapezium

$$\frac{(a+b) \times h}{2}$$



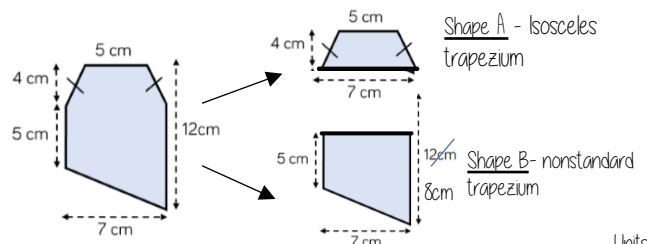
Why?



- Two congruent trapeziums make a parallelogram
- New length  $(a + b) \times \text{height}$
- Divide by 2 to find area of one

### Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



$$\text{Shape A} + \text{Shape B} = \text{total area}$$

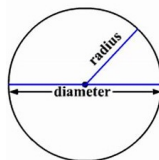
$$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 7}{2} = 24 + 45.5 = 69.5 \text{ cm}^2$$

Units

### Area of a circle (Non-Calculator)

Read the question – leave in terms of  $\pi$  or if  $\pi \approx 3$  (provides an estimate for answers)

Area of a circle  
 $\pi \times \text{radius}^2$



Diameter = 8cm  
 $\therefore$  Radius = 4cm

$$\begin{aligned} \pi \times \text{radius}^2 \\ = \pi \times 4^2 \\ = \pi \times 16 \\ = 16\pi \text{ cm}^2 \end{aligned}$$

Find the area of one quarter of the circle



Radius = 4cm

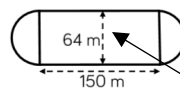
$$\begin{aligned} \text{Circle Area} &= 16\pi \text{ cm}^2 \\ \text{Quarter} &= 4\pi \text{ cm}^2 \end{aligned}$$

### Compound shapes including circles

Circumference  
 $\pi \times \text{diameter}$

Compound shapes are not always area questions  
For Perimeter you will need to use the circumference

#### Spotting diameters and radii



This dimension is also the diameter of the semi circles.

$$\begin{aligned} \text{Arc lengths} &= \pi \times 64 \\ &= 64\pi \end{aligned}$$

Don't need to halve this because there are 2 ends which make the whole circle.

Arc lengths + Straight lengths = total perimeter

$$\begin{aligned} &= 64\pi + 150 + 150 \\ &= (300 + 64\pi) \text{ m} \\ \text{OR} &= 501.1 \text{ m} \end{aligned}$$

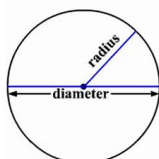
Still remember to split up the compound shape into smaller more manageable individual shapes first

### Area of a circle (Calculator)



SHIFT  $\times 10^{-1}$

Area of a circle  
 $\pi \times \text{radius}^2$



How to get  $\pi$  symbol on the calculator

It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

# YEAR 8 - DEVELOPING GEOMETRY...



## Line symmetry and reflection

### What do I need to be able to do?

By the end of this unit you should be able to:

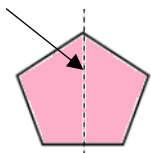
- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

### Keywords

- Mirror line:** a line that passes through the centre of a shape with a mirror image on either side of the line
- Line of symmetry:** same definition as the mirror line
- Reflect:** mapping of one object from one position to another of equal distance from a given line.
- Vertex:** a point where two or more line segments meet
- Perpendicular:** lines that cross at  $90^\circ$
- Horizontal:** a straight line from left to right (parallel to the x axis)
- Vertical:** a straight line from top to bottom (parallel to the y axis)

### Lines of symmetry

Mirror line (line of reflection)



Shapes can have more than one line of symmetry...  
This regular polygon (a regular pentagon has 5 lines of symmetry)



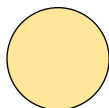
Rhombus  
two lines of symmetry

Parallelogram

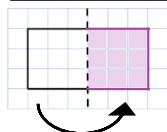
No lines of symmetry



A circle has an infinite amount of lines of symmetry

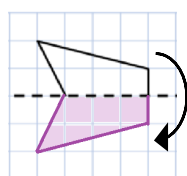


### Reflect horizontally/ vertically (1)



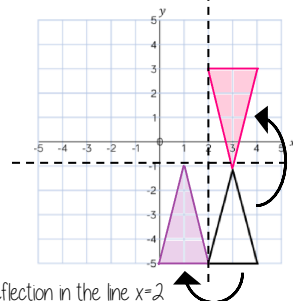
Reflection in a vertical line

Note: a reflection doubles the area of the original shape



Reflection in a horizontal line

Reflection on an axis grid

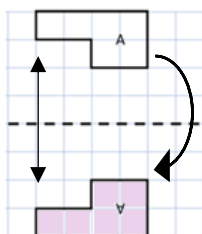


Reflection in the line  $y=-2$

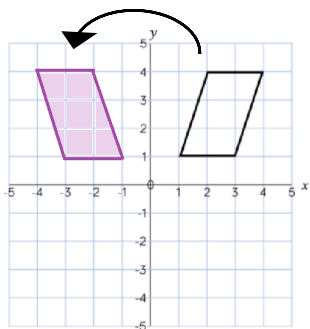
Reflection in the line  $x=2$

### Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



Reflection in the line y axis – this is also a reflection in the line  $x=0$



Lines parallel to the x and y axis

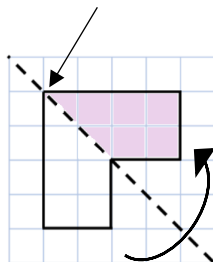
REMEMBER

Lines parallel to the x-axis are  $y = \dots$

Lines parallel to the y-axis are  $x = \dots$

### Reflect Diagonally (1)

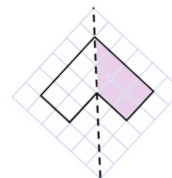
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

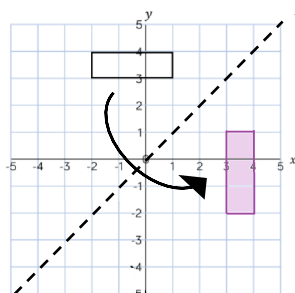


Drawing perpendicular lines

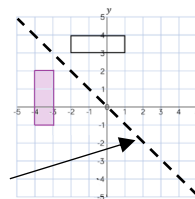
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

### Reflect Diagonally (2)

This is the line  $y = x$  (every y coordinate is the same as the x coordinate along this line)



This is the line  $y = -x$   
The x and y coordinate have the same value but opposite sign



Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

# YEAR 8 - REASONING WITH DATA...



## The data handling cycle

### What do I need to be able to do?

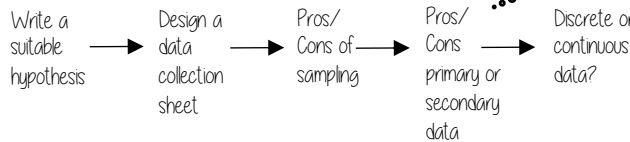
#### By the end of this unit you should be able to:

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret multiple bar charts
- Draw and interpret line graphs
- Represent and interpret grouped quantitative data
- Find and interpret the range
- Compare distributions

### Keywords

- Hypothesis:** an idea or question you want to test
- Sampling:** the group of things you want to use to check your hypothesis
- Primary Data:** data you collect yourself
- Secondary Data:** data you source from elsewhere e.g. the internet/ newspapers/ local statistics
- Discrete Data:** numerical data that can only take set values
- Continuous Data:** numerical data that has an infinite number of values (often seen with height, distance, time)
- Spread:** the distance/ how spread out/ variation of data
- Average:** a measure of central tendency – or the typical value of all the data together
- Proportion:** numerical relationship that compares two things

### Set up a statistical enquiry



#### Features of a data collection sheet

Data Title	Tally	Frequency

Grouped or ungrouped categories

Total number of that group observed

### Design and criticise a questionnaire

**The Question** - be clear with the question - don't be too leading/ judgemental

e.g. How much pocket money do you get a week?

**Responses** - do you want closed or open responses? - do any options overlap? - Have you an option for all responses?

Zero option  £0  £0.01 - £2  £2.01 - £4  more than £4 More option

NOTE: For responses about continuous data include inequalities  $< x \leq$

### Pictograms, bar and line charts

#### Pictogram

Language	Number of people
French	8
Spanish	6
German	2

1 circle = 4 people

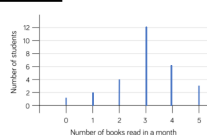
- Need to remember a key
- Visually able to identify mode

#### Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

#### Line Chart



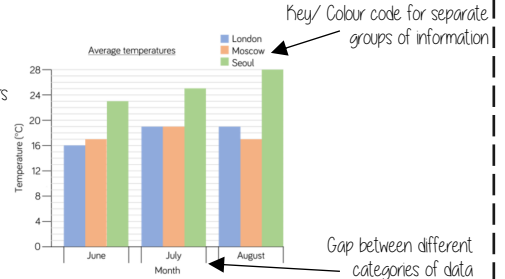
- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Represents quantitative data

### Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



Key/ Colour code for separate groups of information

Gap between different categories of data

### Draw and interpret Pie Charts

Remember a circle has 360°

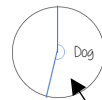
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$$\frac{32}{60} \times 360 = 192^\circ$$



Use a protractor to draw This is 192°

**Multiple method**  
As 60 goes into 360 - 6 times  
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

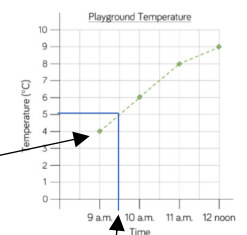
Represents quantitative, discrete data

### Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points

Line graphs do not need to start from 0

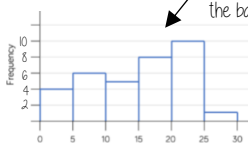
More than one piece of data can be plotted on the same graph to compare data



It is possible to make estimates from the line e.g. temperature at 9.30am is 5°C

### Grouped quantitative data

Time (minutes)	Frequency
$0 \leq t < 5$	4
$5 \leq t < 10$	6
$10 \leq t < 15$	5
$15 \leq t < 20$	8
$20 \leq t < 25$	10
$25 \leq t < 30$	1



This is a frequency diagram There are no gaps between the bars

Grouping the data is useful if there is a large spread of data to begin with

"More than or equal to 25 and less than 30 minutes"

The use of inequalities shows that this will be a frequency diagram

### Find and interpret the range

The range is a measure of spread

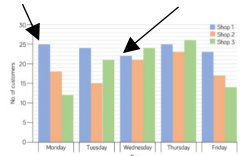
A smaller range means there is less variation in the results - it is more consistent data

A range of 0 means all the data is the same value

Shop 1 has the smallest range - this indicates it has a more consistent flow of customers each week

Difference between the biggest and smallest values

Shop 1 highest value      Shop 1 lowest value



Range of customers =  $25 - 22 = 3$  (Shop 1)

# YEAR 8 - REASONING WITH DATA...



## Measures of location

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use mean, median and mode
- Choose the most appropriate average
- Identify outliers
- Compare distributions using averages and range

### Keywords

- Spread:** the distance/ how spread out/ variation of data
- Average:** a measure of central tendency – or the typical value of all the data together
- Total:** all the data added together
- Frequency:** the number of times the data values occur
- Represent:** something that shows the value of another
- Outlier:** a value that stands apart from the data set
- Consistent:** a set of data that is similar and doesn't change very much

### Mean, Median, Mode

#### The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values) 55

Divide the overall total by how many pieces of data you have  $55 \div 5$

Mean = 11

#### The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, 8, 11, 24

Median = 8

NOTE: if there is no single middle value find the mean of the two numbers left

#### The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if the data is ordered first

4, 8, 8, 11, 24

Mode = 8

### Choosing the appropriate average

The average should be a representative of the data set – so it should be compared to the set as a whole – to check if it is an appropriate average

Here are the weekly wages of a small firm

£240   £240   £240   £240   £240  
£260   £260   £300   £350   £700

Which average best represents the weekly wage?

The Mean = £307

The Median = £250

The Mode = £240

Put the data back into context

Mean/Median – too high (most of this company earn £240)

Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members – their salary doesn't represent the average weekly wage of the majority of employees

### Identify outliers

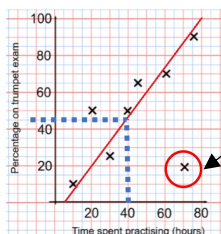
Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean. They have less impact on the median and the mode

Sometimes it is best to not use an outlier in calculations

Height in cm  
152 150 142 158 182 151 153 149 156 160 151 144

Where an outlier is identified try to give it some context. This is likely to be a taller member of the group. Could be an older student or a teacher?



Outliers can also be identified graphically e.g. on scatter graphs

### Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency

Here are the number of runs scored last month by Lucy and James in cricket matches

Lucy: 45, 32, 37, 41, 48, 35  
James: 60, 90, 41, 23, 14, 23

Lucy

Mean: 39.6 (1dp), Median: 38, Mode: no mode, Range: 16

James

Mean: 41.8 (1dp), Median: 32, Mode: 23, Range: 76

James has two extreme values that have a big impact on the range

"James is less consistent than Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"